EYESHOTS

Heterogeneous 3D Perception across Visual Fragments

CORTICAL ARCHITECTURES FOR DYNAMIC STEREOPSIS

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DIBE – University of Genoa

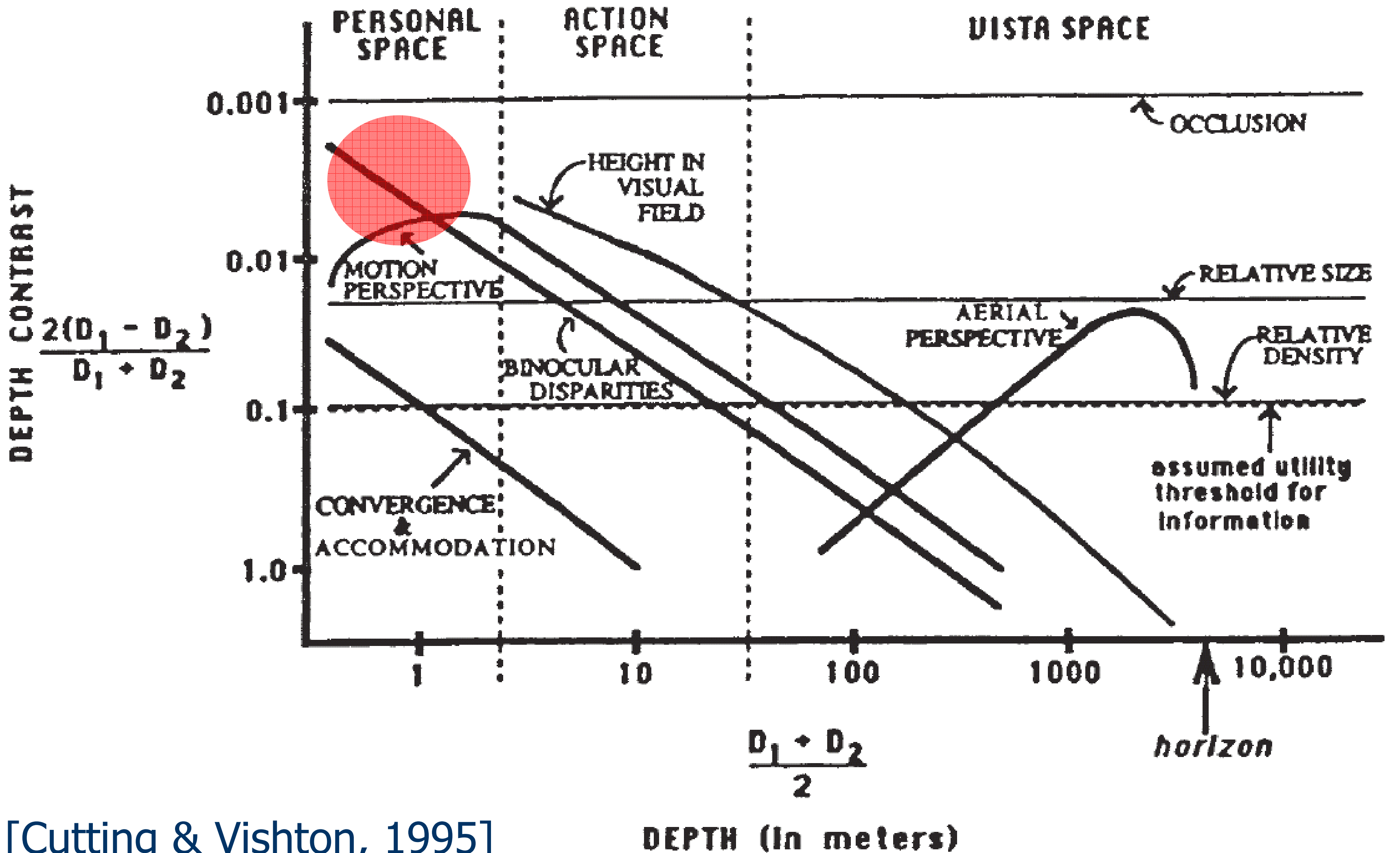
PSPC The Physical Structure of Perception and Computation

Dibe

PRESENTATION OUTLINE

- The computational theory of stereo vision
  - Geometrical issues
  - Disparities as measures on the visual signals
  - The active vision perspective
- Cortical disparity detectors
  - Hierarchical processing of depth information
  - The binocular energy models
  - 2nd order disparities (pb and neuromorphic solutions)
- Work in progress under EYESHOTS' perspective

JNDs FOR DIFFERENT DEPTH CUES



[Cutting & Vishton, 1995]

STEREOSCOPIC VISION

- The most important mechanism for assessing depth in human vision. First enunciated in 1838 by Sir Charles Wheatstone, depends on the slight differences in the two pictures projected on the retinas (binocular disparity).
  - Effect: the sensation of 3D is caused by the images falling on the fovea on one eye and its Panum's area in the other. Stereopsis allows us to appreciate depth and judge distances
  - Cause: the process in visual perception leading to perception of stereoscopic depth.

CORRESPONDING POINTS AND EPIPOLAR LINES

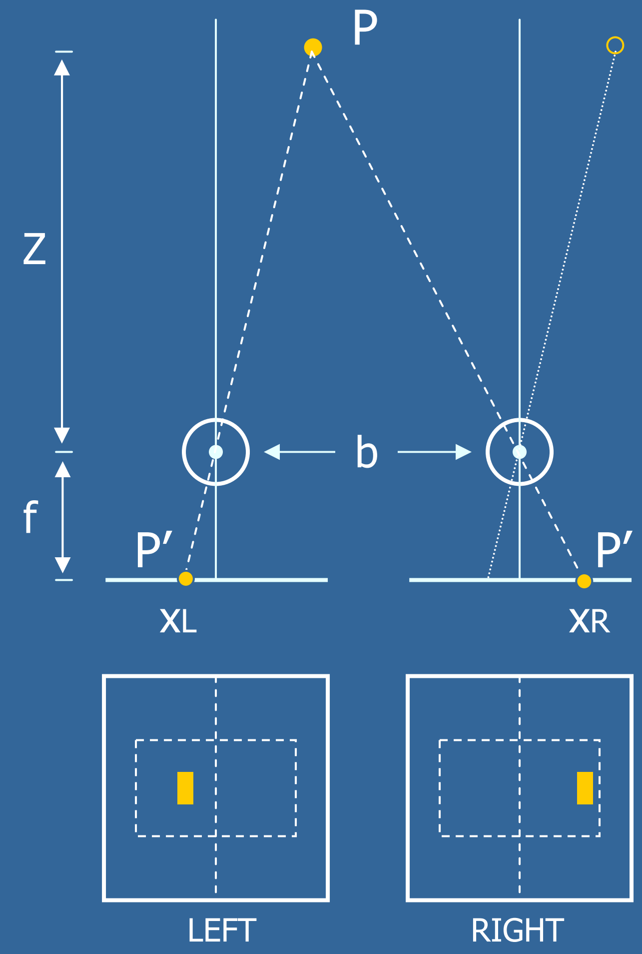
<u>The case of parallel optical axes</u>

- The point P is imaged in the left and right images with different horizontal coordinates

$$\delta = x_L - x_R$$

- Application of parallel proportionality theorem results in

$$\delta = -f\frac{b}{Z}$$

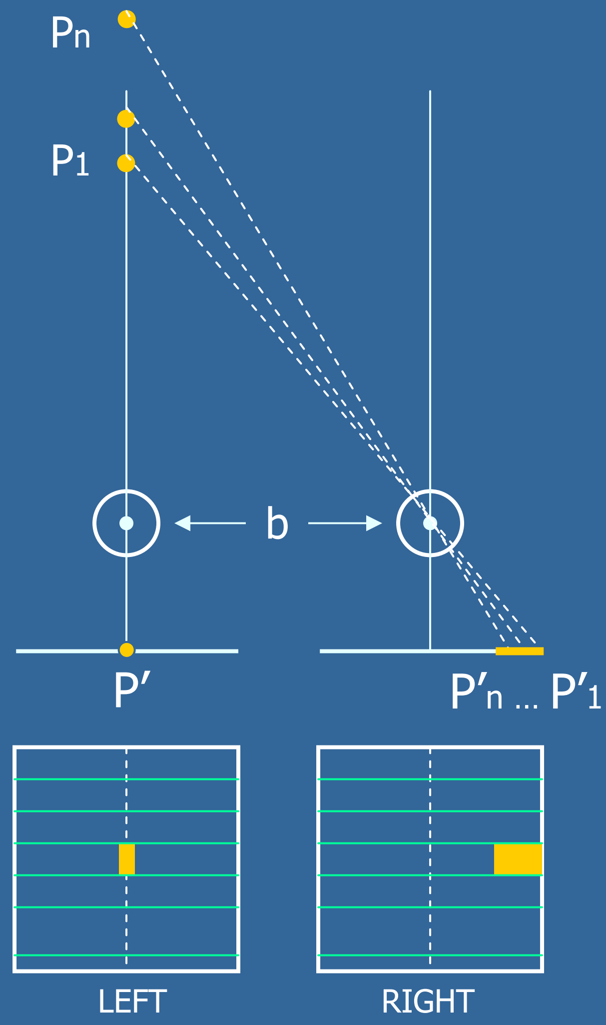


CORRESPONDING POINTS AND EPIPOLAR LINES

<u>The case of parallel optical axes</u>

- For any point in one eye the set of possible matches defines a line in the other eye

- The *epipolar line* is the locus of all possible matches in the right eye for point P in the left eye



MEASURES IN THE PLENOPTIC SPACE

- What can be potentially seen?
- What information about the world is contained in the light filling a region of space?

→ The *Plenoptic Function* [Adelson & Bergen, 1991] is an idealized concept representing the whole set of images that can be collected by translating the eye through a range of viewing positions

$$P(x, y, t, V_X, V_Y, V_Z)$$

- Interactions with objects take place through measures of the pattern of light rays reflected by objects

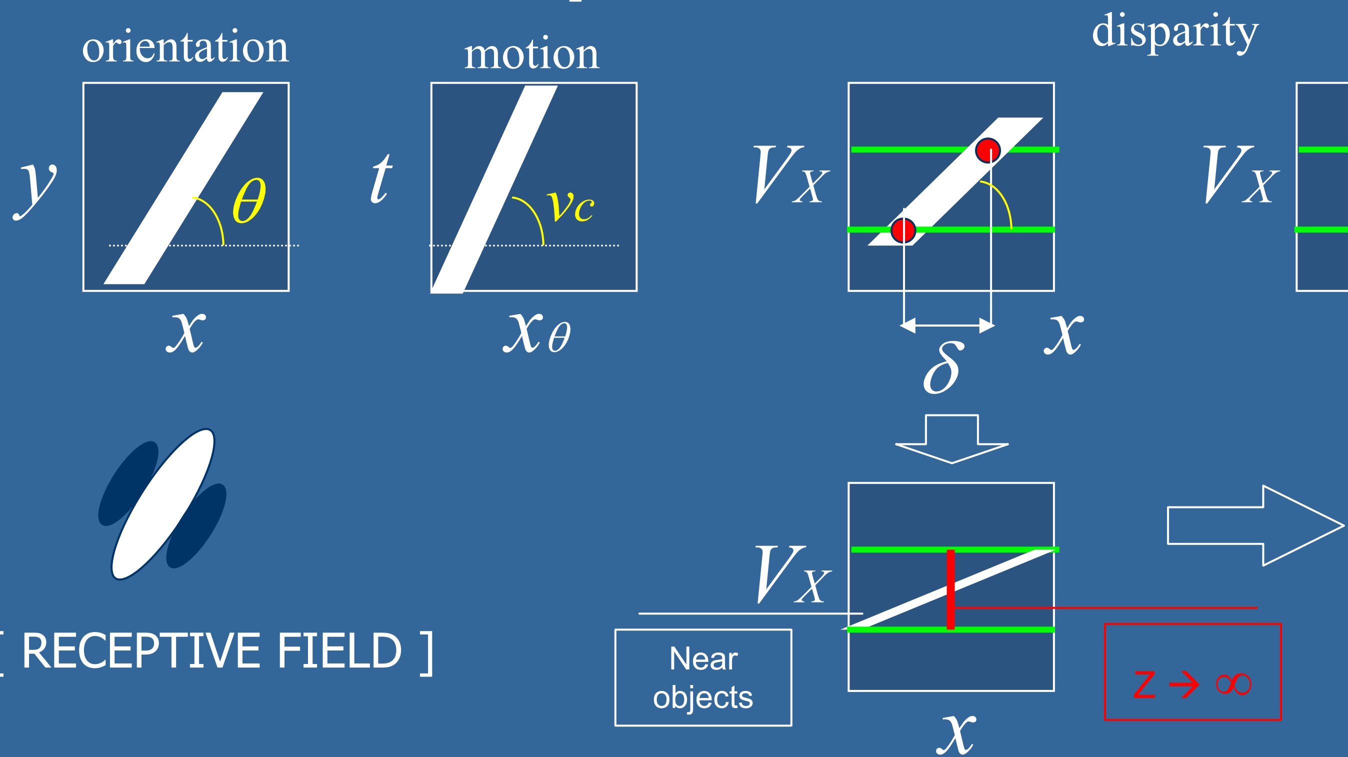
The plenoptic function acts as a communication link between physical objects and their corresponding retinal images.

MEASURES IN THE PLENOPTIC SPACE

- Features as measures of local (oriented) discontinuities in the Plenoptic Function

$$P(x, y, t, V_X, V_Y, V_Z)$$

[LOCAL NEIGHBORHOODS]



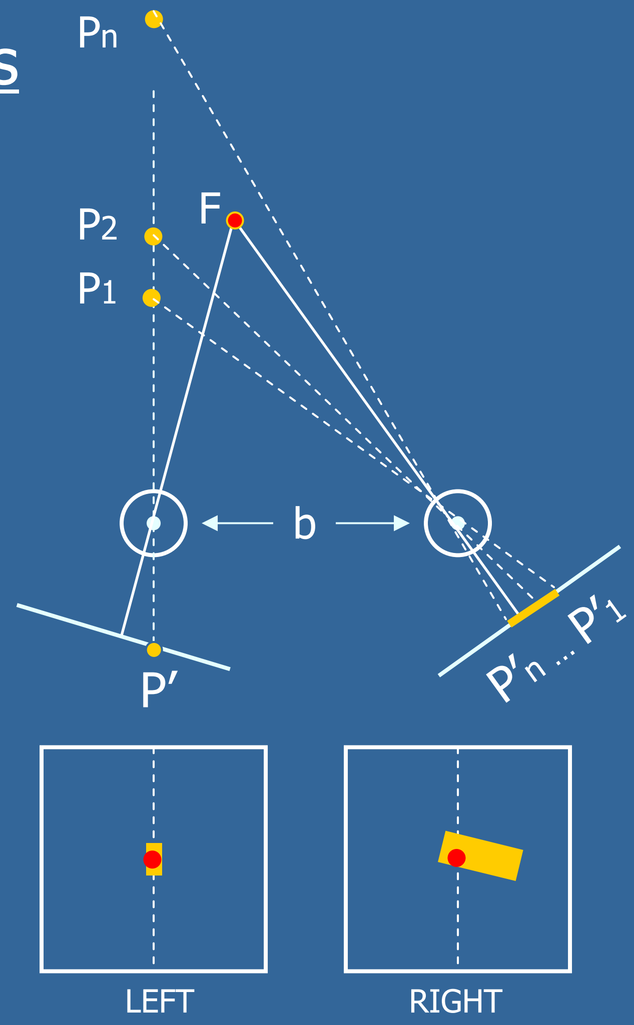
[RECEPTIVE FIELD]

HORIZONTAL DISPARITY TUNING

CORRESPONDING POINTS AND EPIPOLAR LINES

<u>The case of convergent optical axes</u>

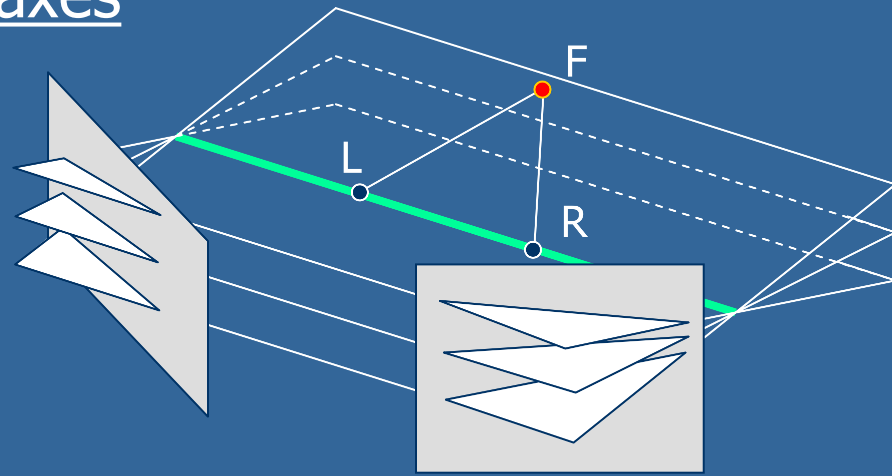
- In human stereopsis the optical axes usually intersect in a common fixation point F
- Deviations from primary position rotate the epipolar lines and vertical disparities (VD) become possible



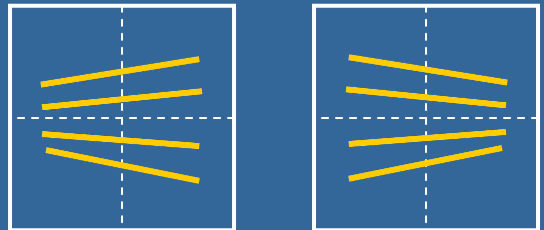
CORRESPONDING POINTS AND EPIPOLAR LINES

The case of convergent optical axes

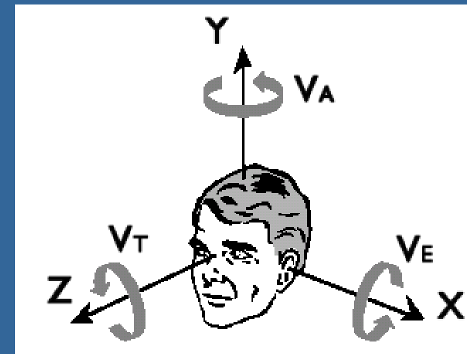
- For verging systems, epipolar lines are slanted. HD is therefore associated with a predictable VD



- Epipolar lines can be used to estimate vergence angle from images.



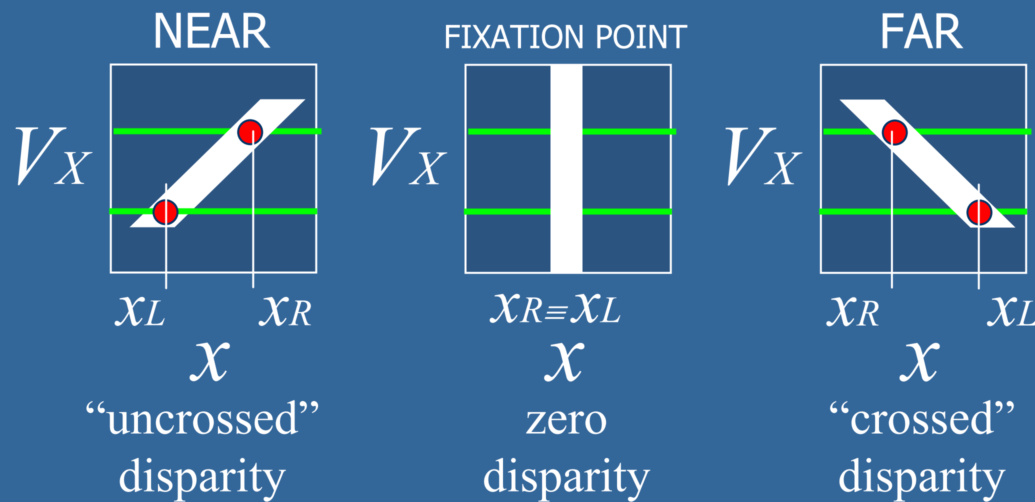
ACTIVE MEASURES IN THE PLENOPTIC SPACE



- Rotations of the eye(s) in the orbit(s) affect the way we visually interact with the external world

$$P(x, y, t, \underbrace{V_X, V_Y, V_Z}_{\text{FIXED}}, V_A, V_E, V_T)$$

ACTION

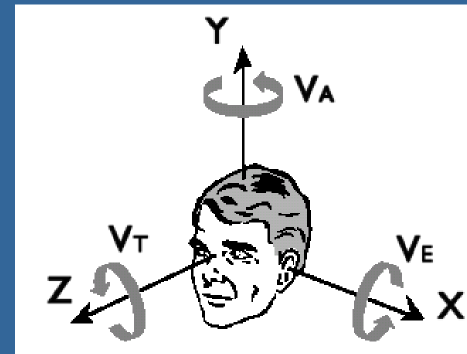


ROLL → $V_T(t)$

TILT → $V_E(t)$

PAN → $V_A(t)$

ACTIVE MEASURES IN THE PLENOPTIC SPACE



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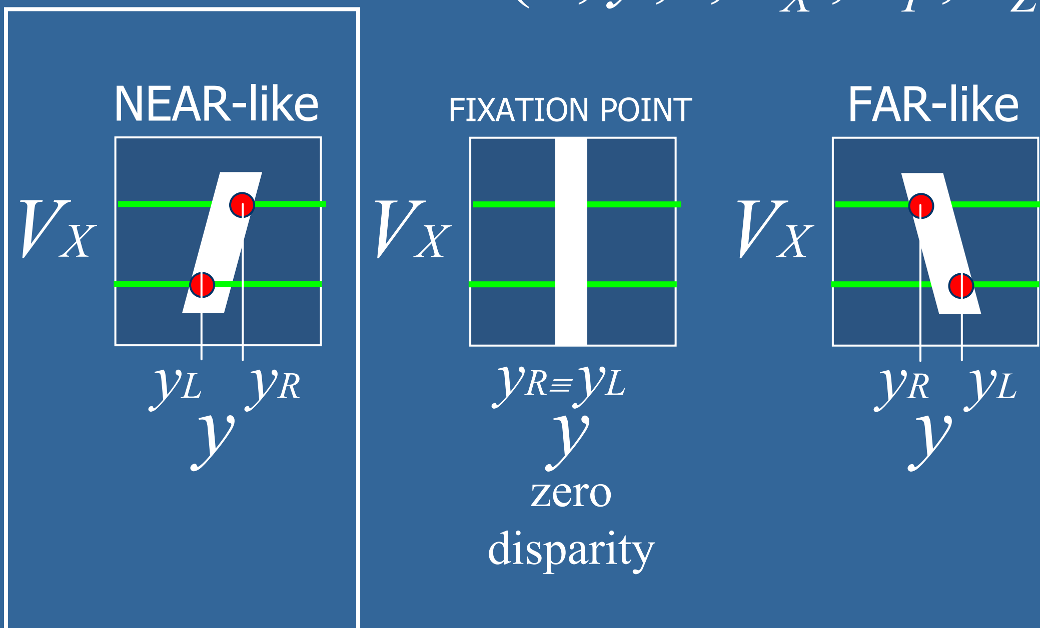

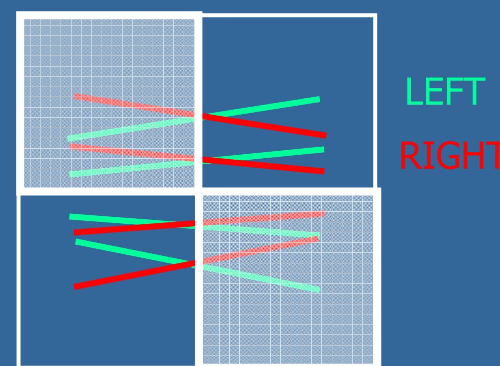

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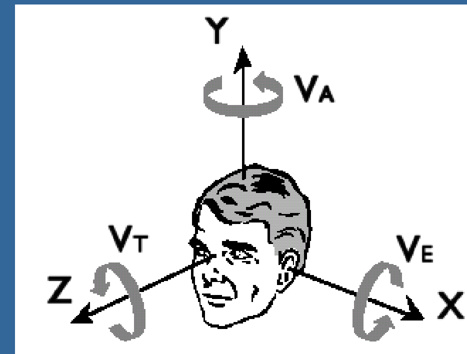

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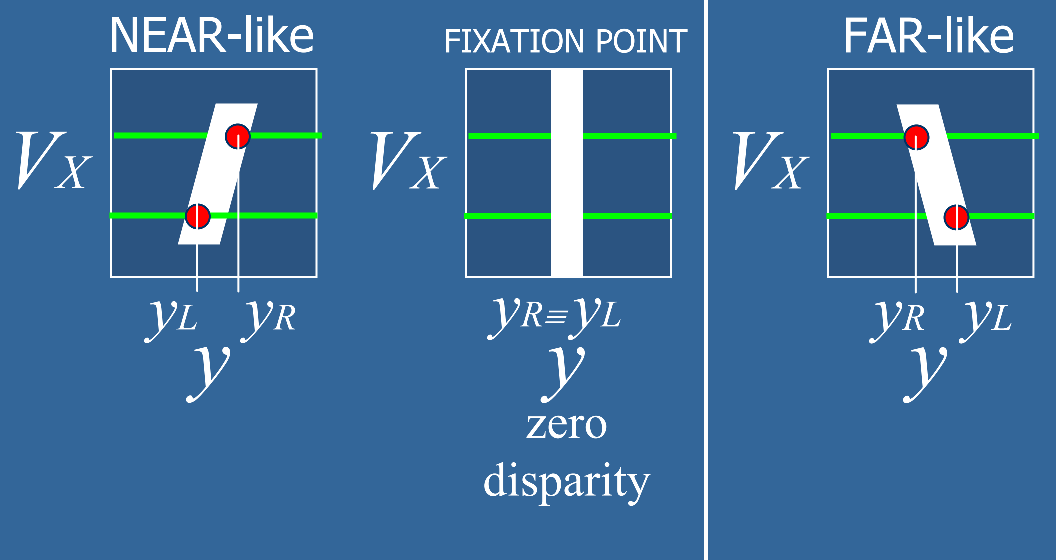

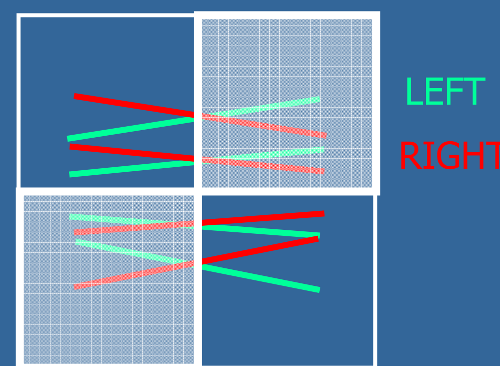

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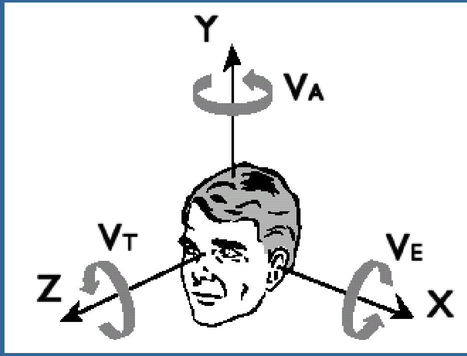

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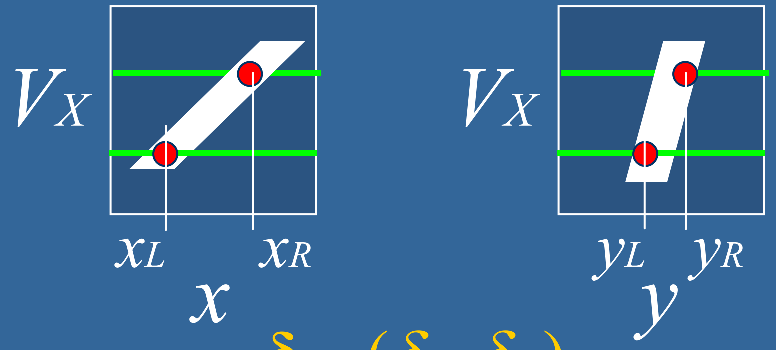


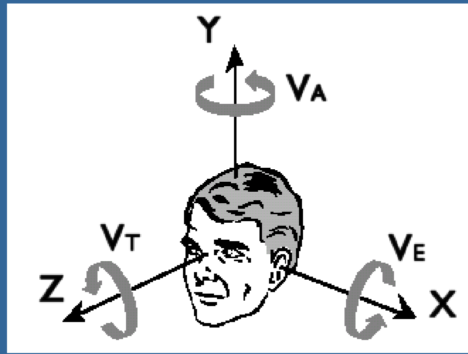
$$\boldsymbol{\delta}=(\delta_x,\delta_y)$$

where $\delta_x = x_L - x_R$

$\delta_y = y_L - y_R$

Oblique disparity (δ_x,δ_y) as the vector difference in positions of identified corresponding points in the left and right eyes, each measured with respect to the point of fixation as origin.

ACTIVE MEASURES IN THE PLENOPTIC SPACE



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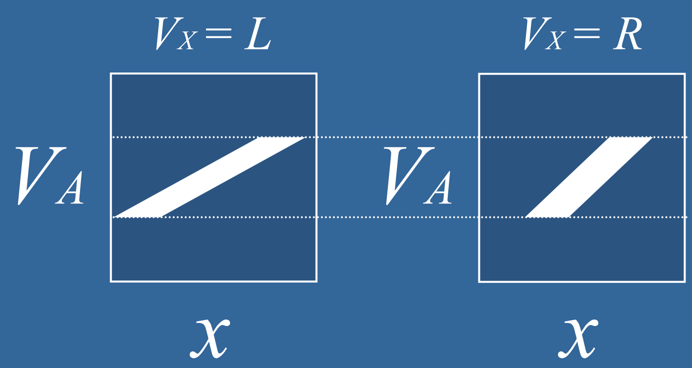


→ Interocular velocity differences

ACTIVE MEASURES IN THE PLENOPTIC SPACE

- 1st-order disparity measures
  - Position disparity
- 2nd-order disparity measures
  - Motion disparity (interocular velocity differences, motion-in-depth)
  - Orientation disparity

JOINT TUNING

ACTIVE MEASURES IN THE PLENOPTIC SPACE

- The visual signal changes actively!
- How can measures on the visual signal change accordingly (in an active way)?

On a local neighborhood this means that:

- New detectors can be learnt
- New coding/decoding mechanisms occur at population level

→ influences on local detectors and/or on selection/decision processes.

ACTIVE MEASURES IN THE PLENOPTIC SPACE

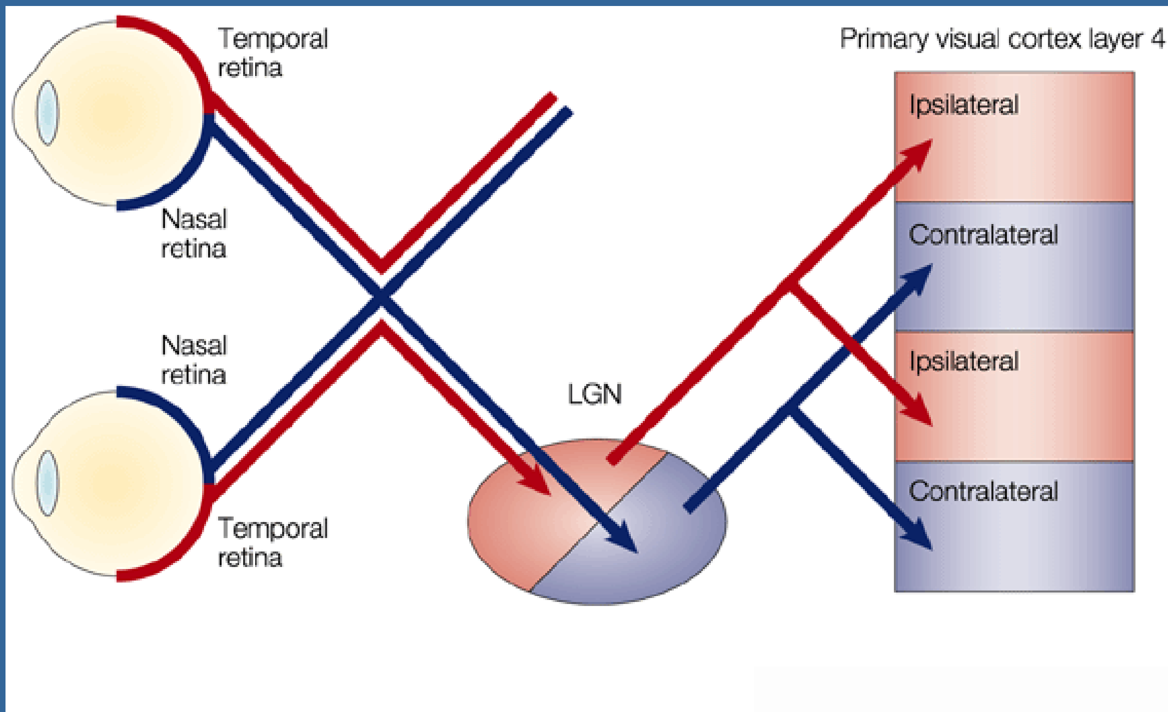
Computational implications:

1) Development of RFs (disparity detectors) from signal of a behaving agent. Stable RFs as soon as the newly learnt behaviour is successful (objective function).
2) New measures of the visual signals. Dynamic characteristics of the visual signal → change detectors (visual feedback)
3) Predictive recruitment of resources (preparation of RF tuning, etc).

CORTICAL DISPARITY DETECTORS

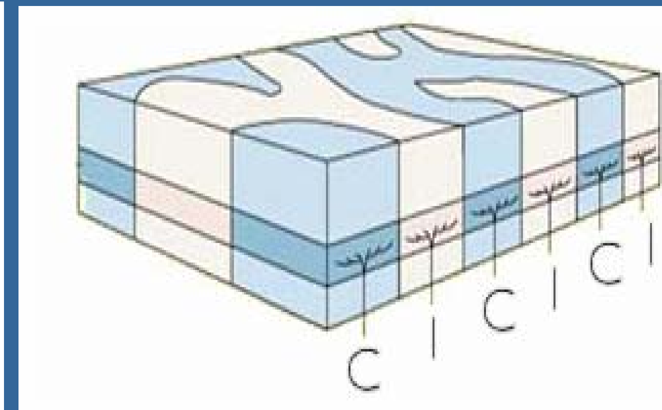
BINOCULAR CONVEGENCE

Cortical mapping of the inputs from the left and the right eyes



[by Katz & Crowley, 2002]

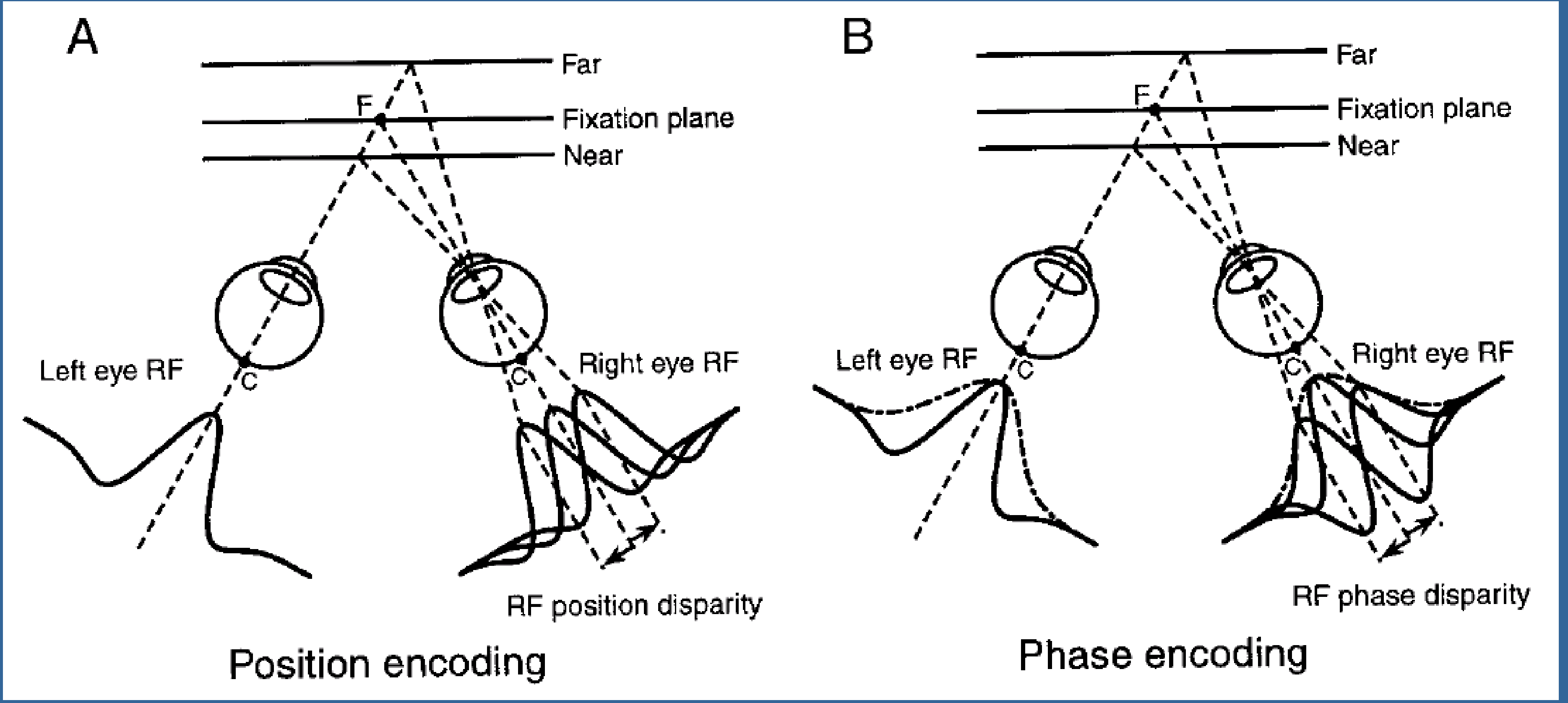
[LeVay et al., 1975]



and

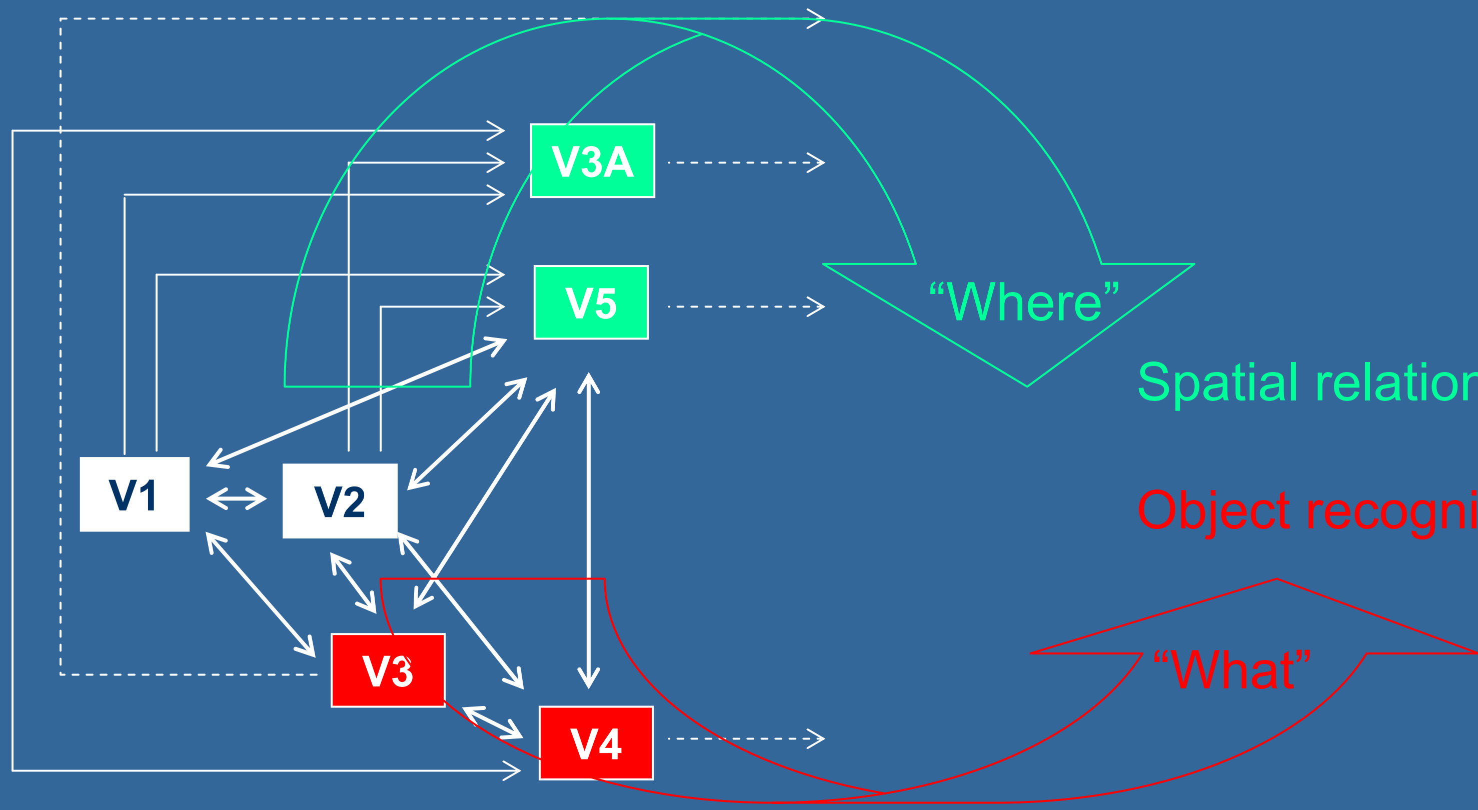
the ocular dominance columns

BASIC DISPARITY DETECTORS



[Anzai et al., 1999]

HIERARCHICAL PROCESSING OF DEPTH INFORMATION (1)



Spatial relationships

Object recognition

[Adapted from Desimone & Ungerleider, 1989]

What is where?

HIERARCHICAL PROCESSING OF DEPTH INFORMATION (2)

[V1] Absolute disparity (early joint stereo-motion tuning)

[V2] Relative disparity
[V2] Disparity edges

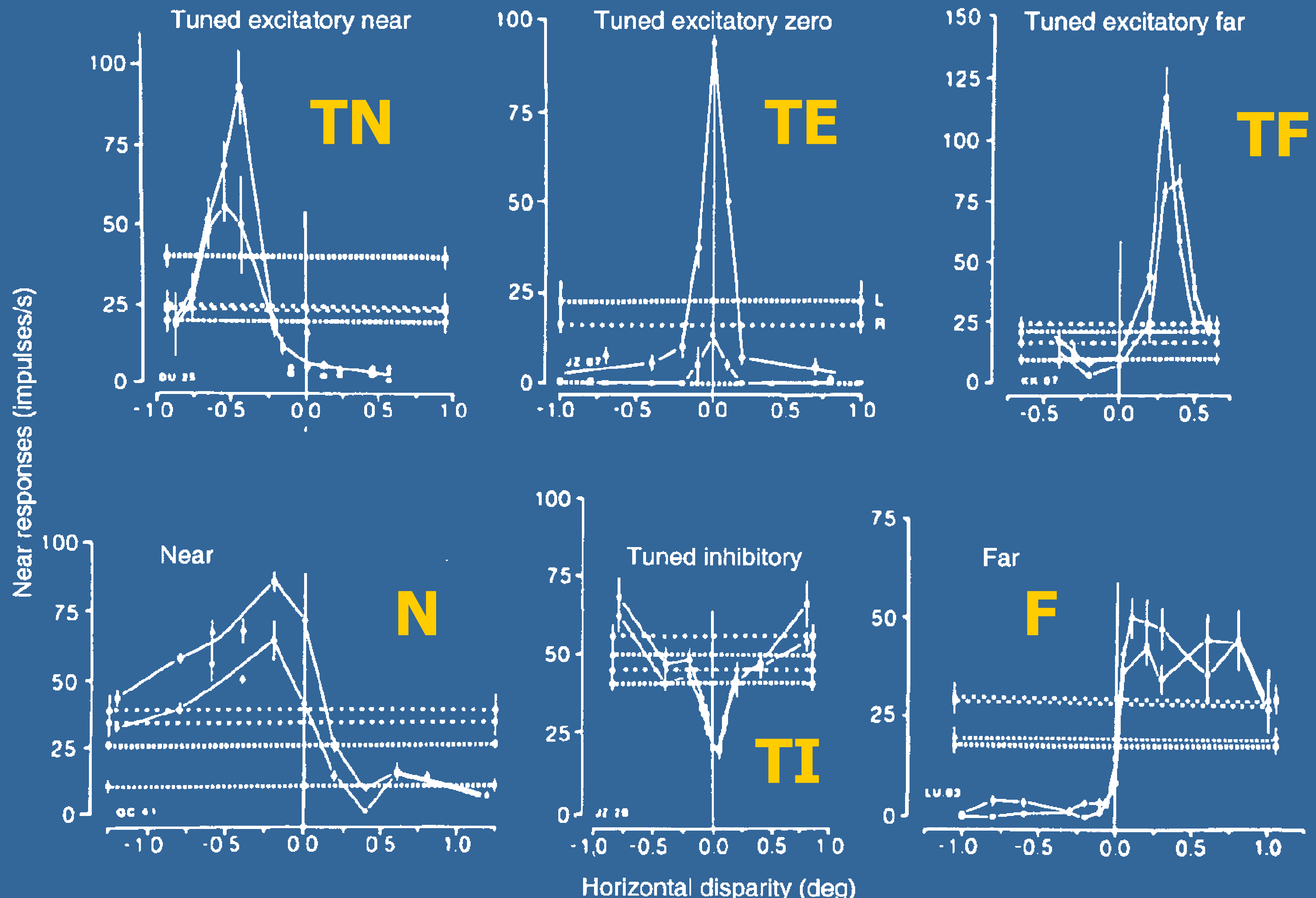
[V3] Higher-order orientation disparities

[V3] Disparity gradients
(projects to the PO, concerned with processing 3D form)

[V4] Orientation of slanted surfaces

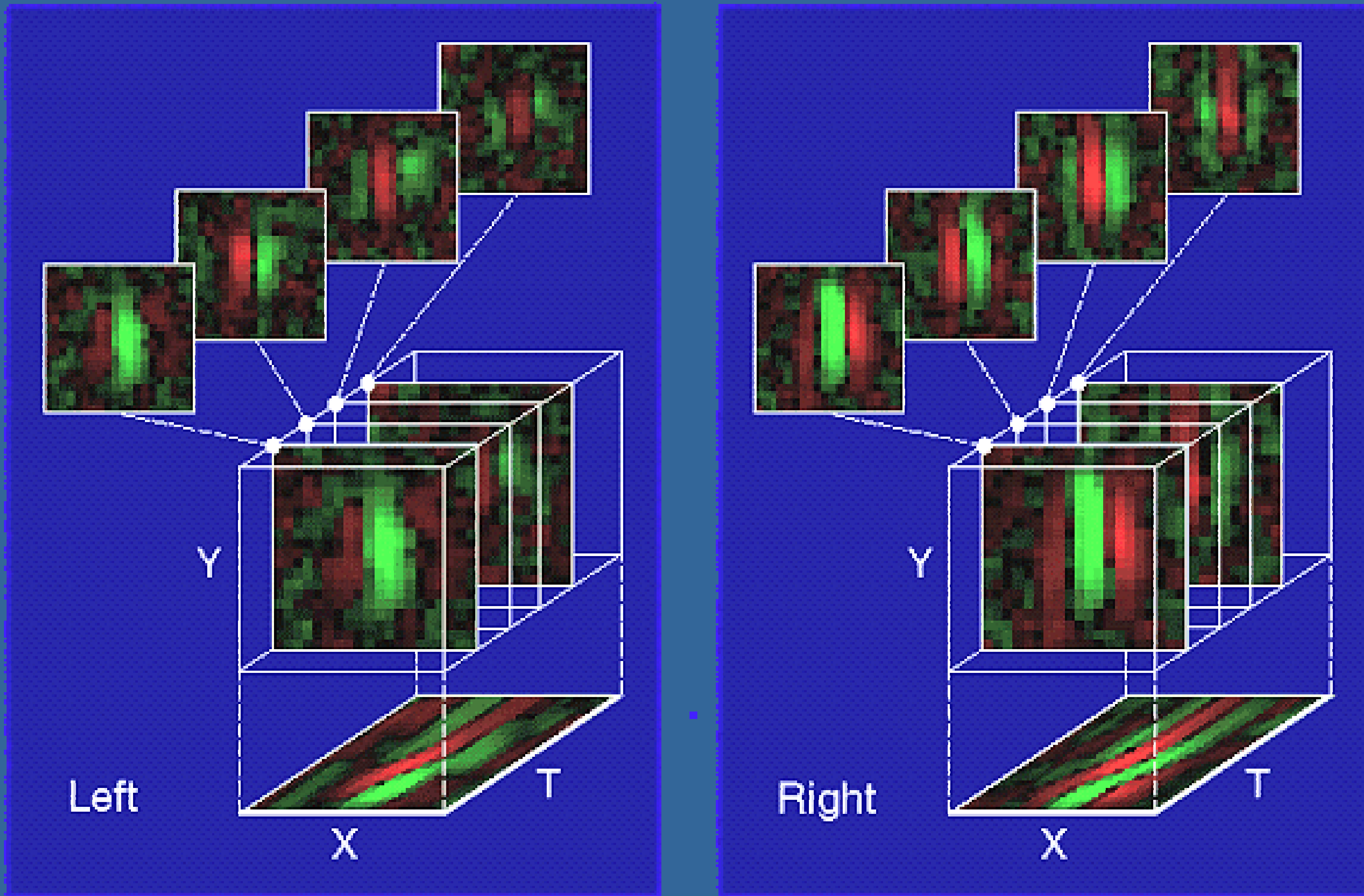
[V5] Joint stereo-motion processing
perceptual segmentation, control of vergence eye movements
(projects to parietal cortex [e.g., LIP])

DISPARITY TUNING



[Poggio, 1991]

SIMPLE CELLS

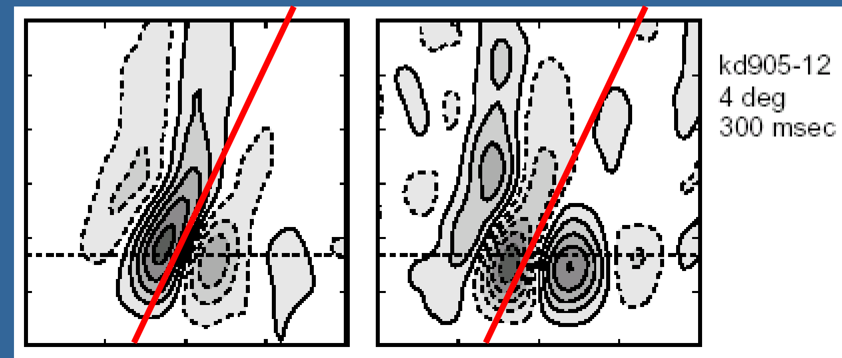


[Ohzawa et al, 1997b]

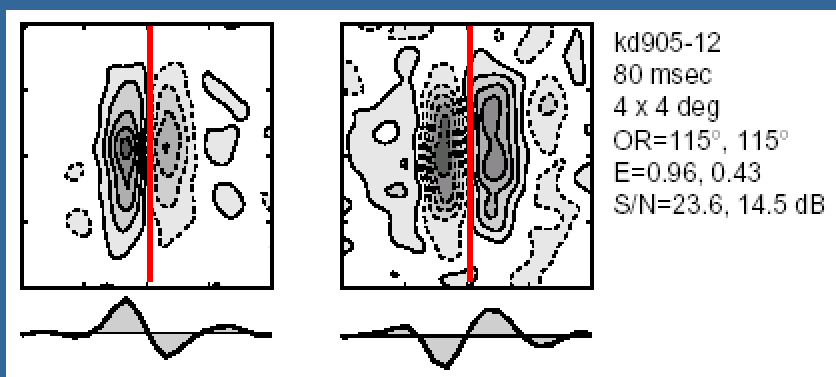
SIMPLE CELLS

- Linear receptive fields
  - Orientation tuning
  - Direction selectivity
  - Binocularity

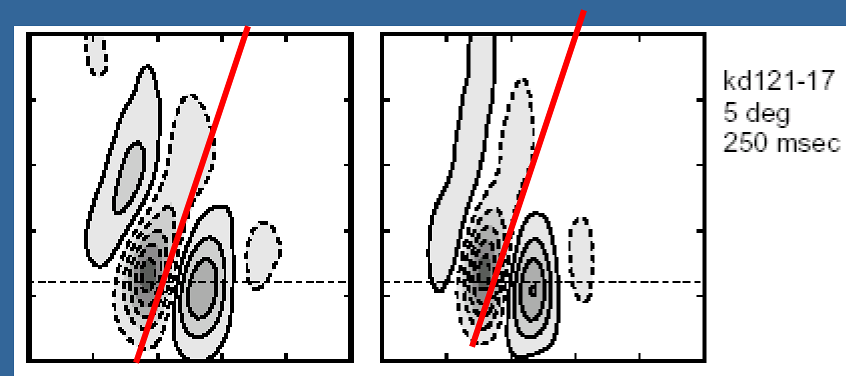
[Ohzawa et al, 1996]



Time [t]



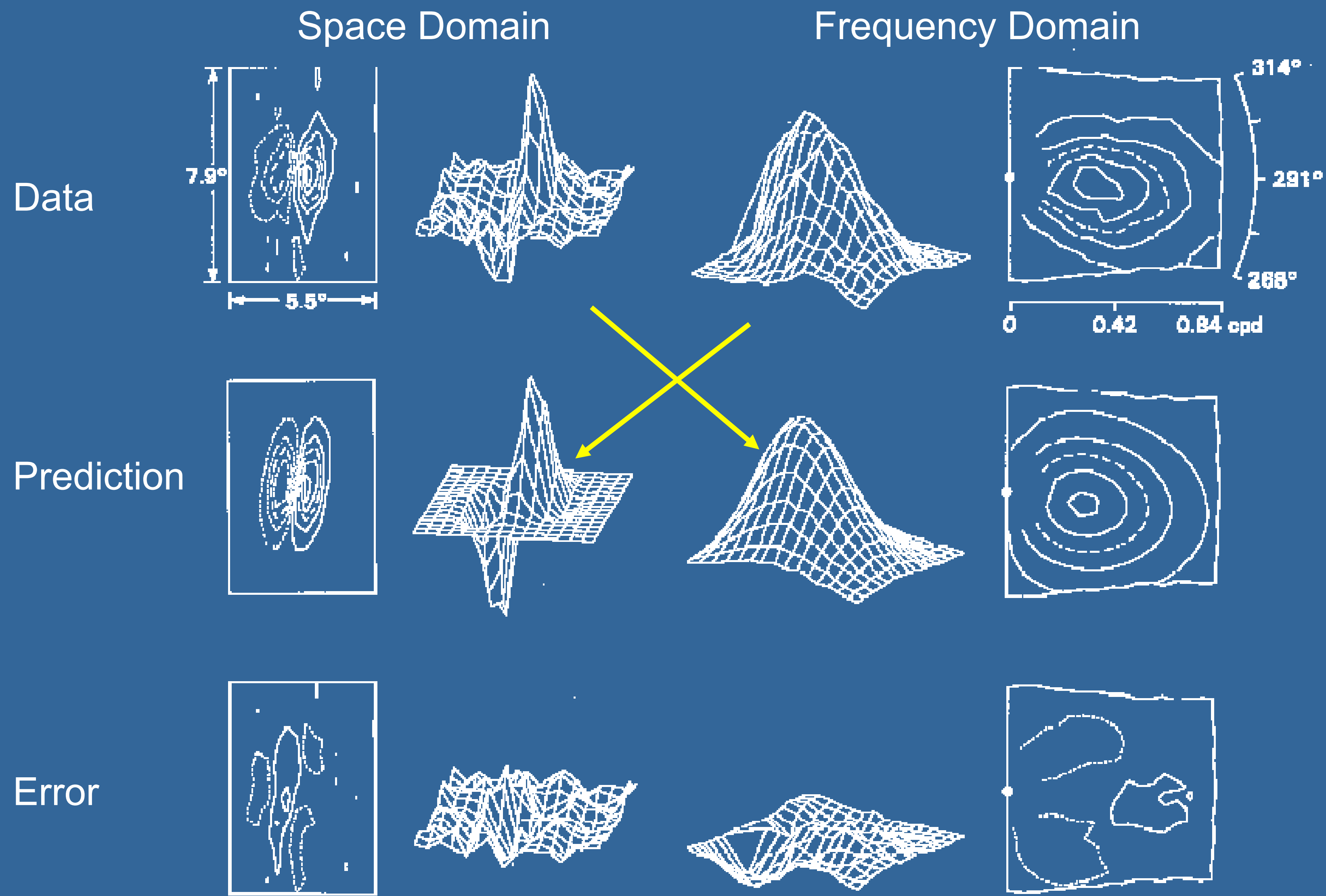
Space [y]



Space [x]

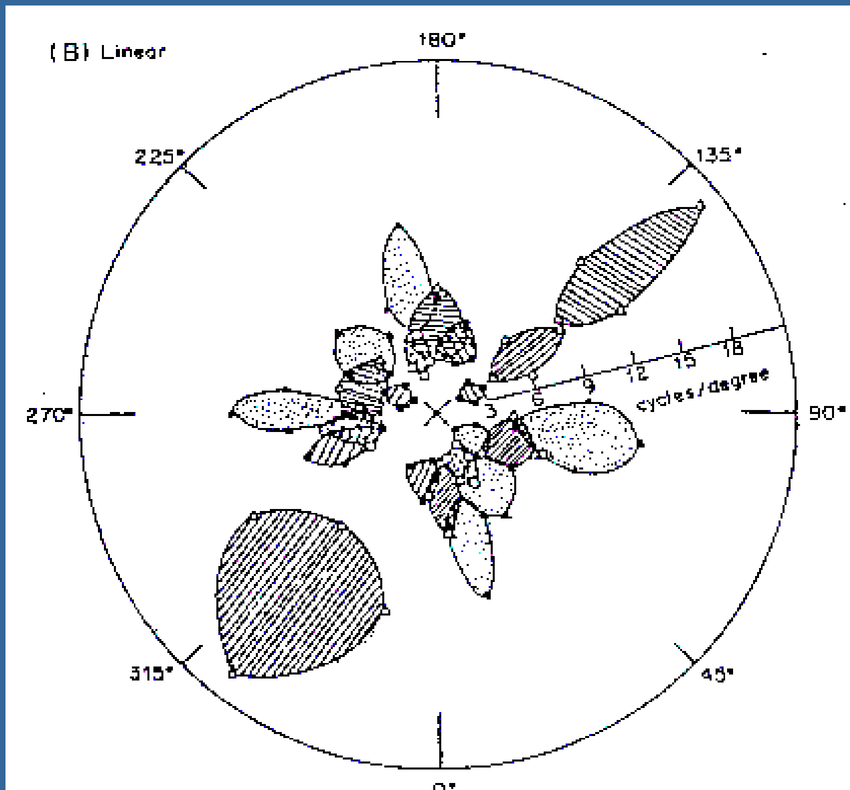
Space [x]

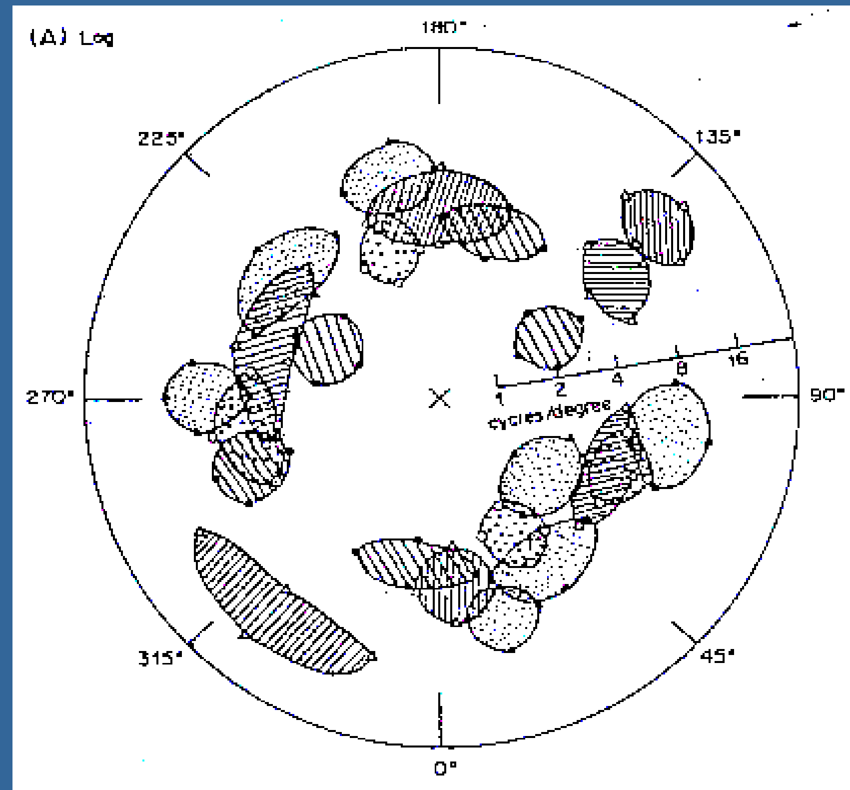
THE LINEARITY ASSUMPTION



[Jones & Palmer, 1987]

SPATIAL FREQUENCY CHANNELS





[De Valois et al., 1982]

[Gabor, 1946]
[Marcelja, 1980]
[Daugman, 1984]

THE 2D GABOR LOGON

$$h(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{x'^2}{2\sigma_x^2}-\frac{y'^2}{2\sigma_y^2}} e^{-j2\pi k_0 x'}$$

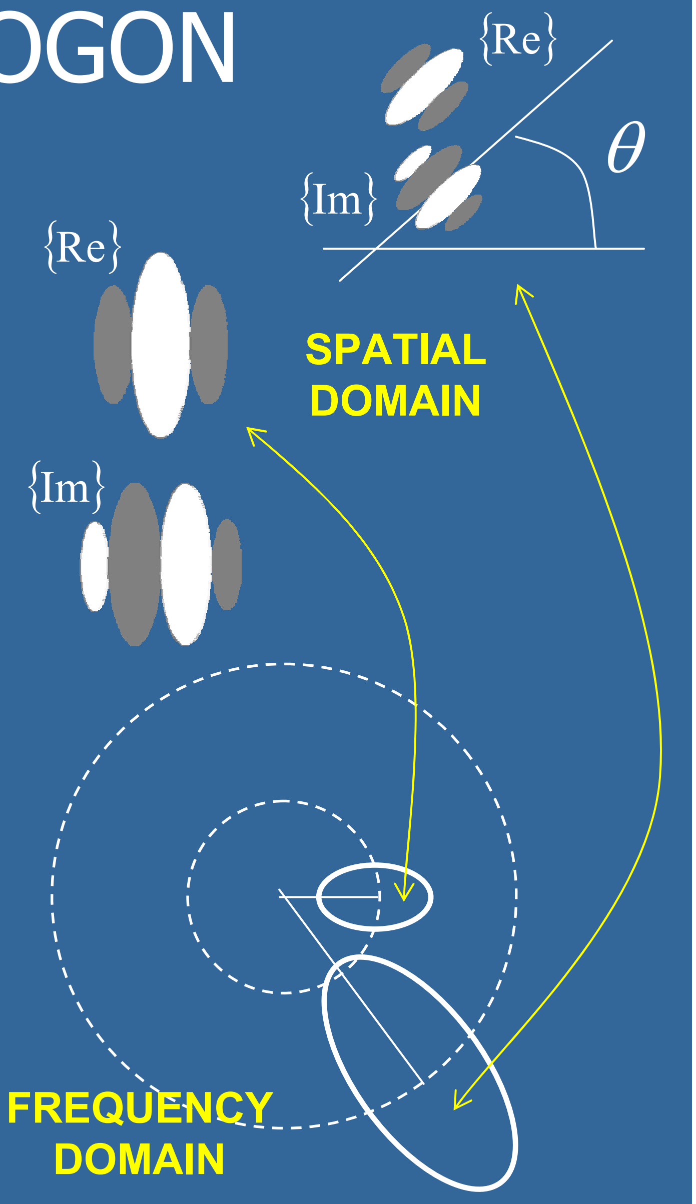
with

$$\begin{cases} x' = (x - x_0)\cos\theta + (y - y_0)\sin\theta \\ y' = -(x - x_0)\sin\theta + (y - y_0)\cos\theta \end{cases}$$

$$\theta = \arctan\frac{k_{0y}}{k_{0x}} + \frac{\pi}{2}$$

$$k_0 = \sqrt{k_{0x}^2 + k_{0y}^2}$$

radial peak frequency



COMPLEX CELLS

- Inherit the tuning of simple cell afferents but *independently* of the phase of the stimulus

L + NL

$$\boxed{L} \longrightarrow (\quad)^2 \longrightarrow$$

cascade model

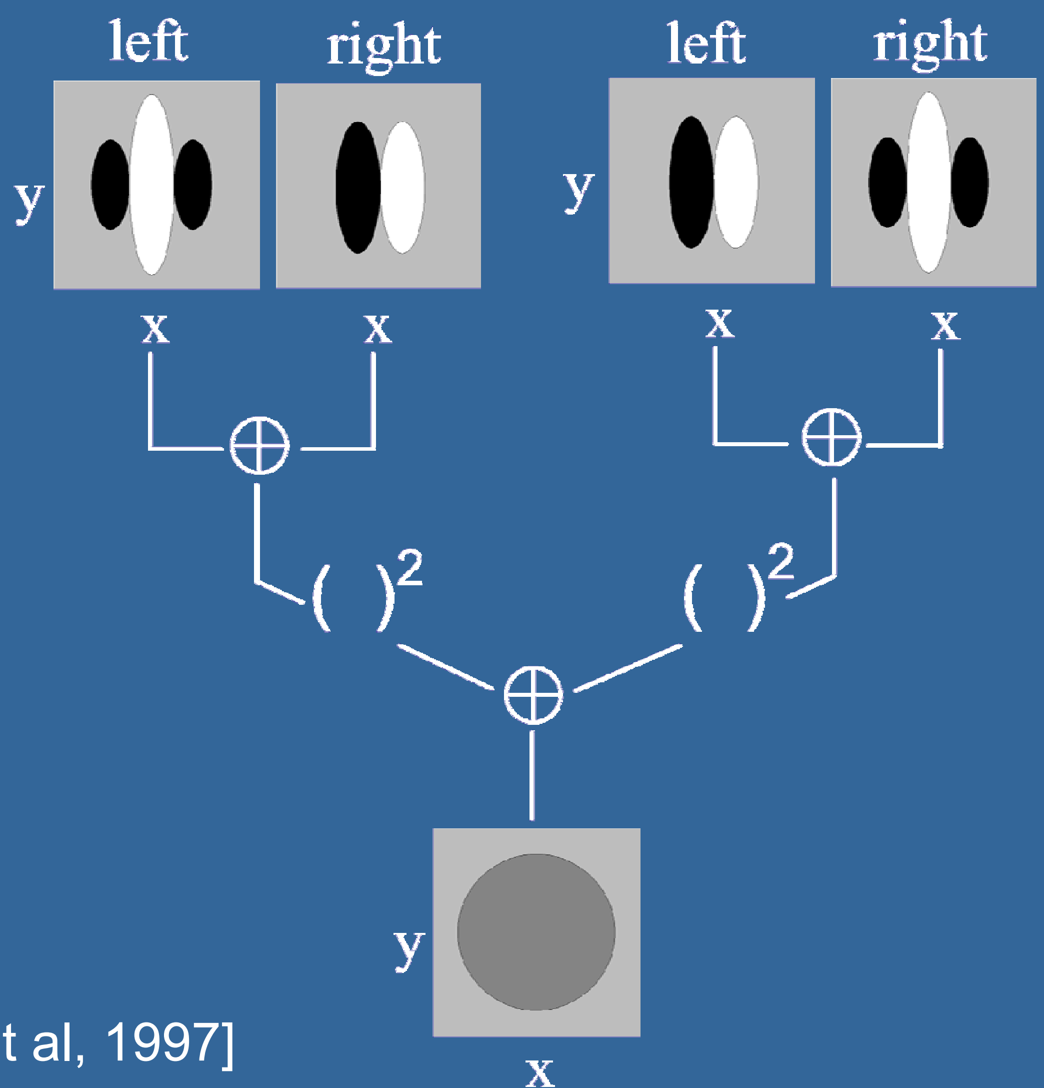
$$\boxed{L_0} \longrightarrow (\quad)^2 \searrow$$

$$\oplus \longrightarrow$$

$$\boxed{L_{90}} \longrightarrow (\quad)^2 \nearrow$$

energy model

BINOCULAR ENERGY MODEL



[Ohzawa et al, 1997]

BINOCULAR ENERGY MODEL

Response of a binocular cell centered in x_0

$$Q(x_0) = \int_a^b \mathbf{g}^L(x - x_0) I^L(x) dx + \int_a^b \mathbf{g}^R(x - x_0) I^R(x) dx$$

where

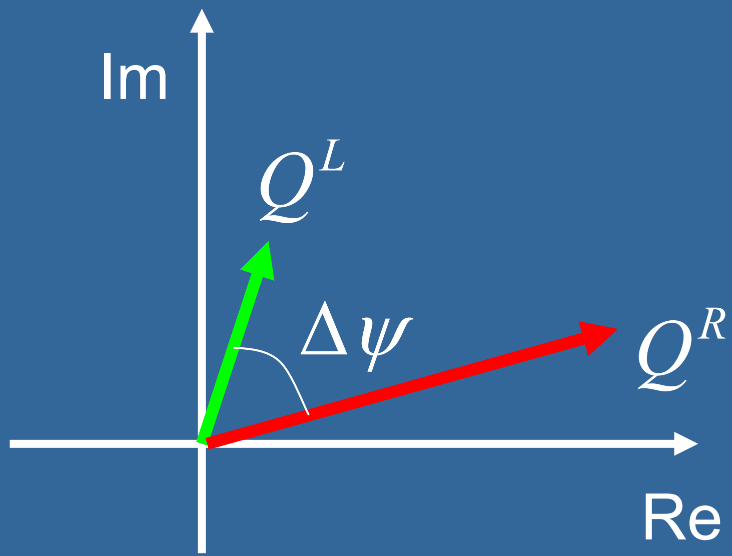
$$\mathbf{g}(x) = g_C(x) + j g_S(x) = G(x) \quad \mathrm{e}^{\mathrm{j}\mathrm{k}_0 (x - x_0)} \mathrm{e}^{\mathrm{j}\psi}$$

Binocular energy unit centered in x_0 **[phase model]**

$$E(x_0) = \left| Q(x_0) \right|^2 = \left| Q^L(x_0) + \mathrm{e}^{\mathrm{j}\Delta\psi} Q^R(x_0) \right|^2$$

where $\Delta\psi = \psi^R - \psi^L$

BINOCULAR ENERGY MODEL



The binocular energy unit maximally responds when $\Delta\psi$ matches the local phase disparity $\Delta\phi$.

>> Tuned to $\delta = \dfrac{\Delta\psi}{k_0}$

Decoding strategy (WTA):

$$\Delta\phi(x) = \arg\max_{\Delta\psi_i} \left| Q^L(x) + \mathrm{e}^{j\Delta\psi_i} Q^R(x) \right|^2$$

Yet, alternative decoding mechanisms can be used (e.g., CoG)

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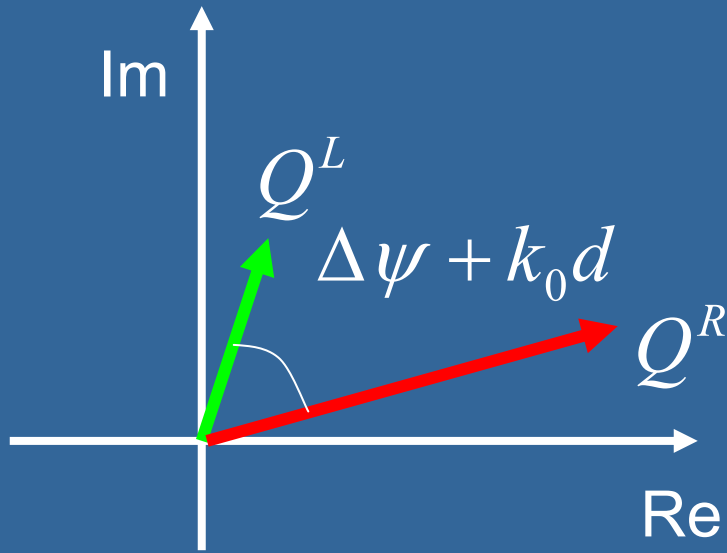
Binocular energy unit centered in x0 **[hybrid model]**

$$E(x_0) = \left|Q(x_0)\right|^2 = \left|Q^L(x_0) + \mathrm{e}^{\mathrm{j}\Delta\psi + \mathrm{j}k_0 d} Q^R(x_0)\right|^2$$

where $\Delta\psi = \psi^R - \psi^L$ and $d = x_0^R - x_0^L$

BINOCULAR ENERGY MODEL



The binocular energy unit maximally responds when $\Delta\psi + d$ matches the local hybrid disparity $\Delta\phi + \Delta x$

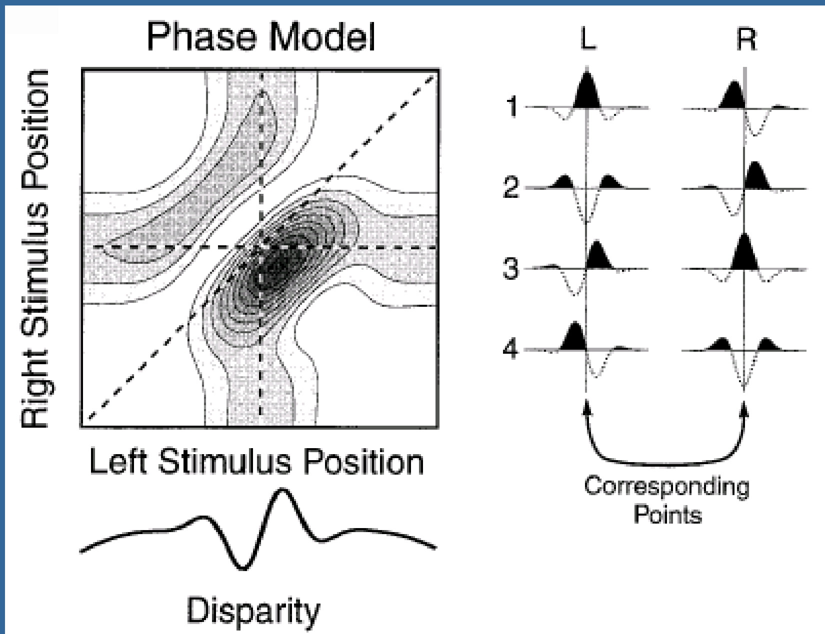
>> <u>Tuned to</u> $\delta = \dfrac{\Delta\psi}{k_0} + d$

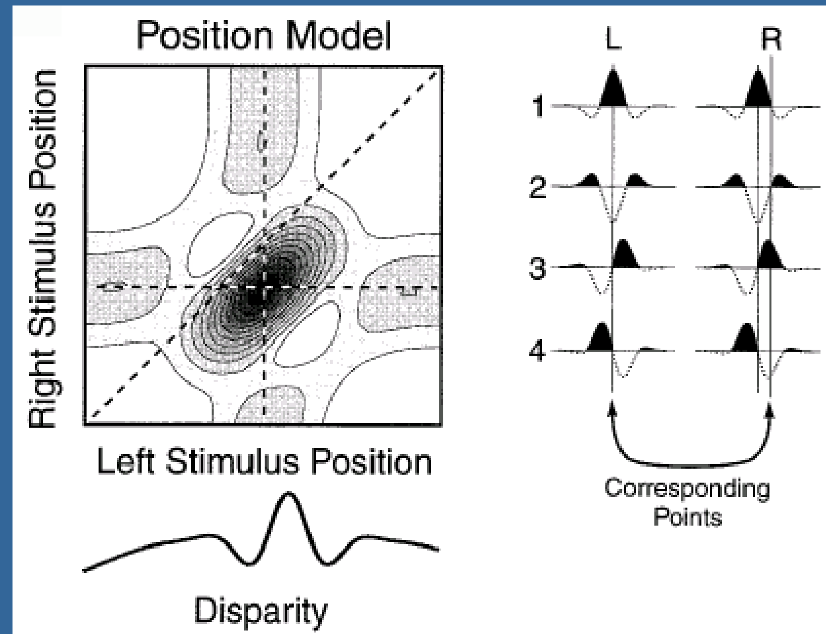
Decoding strategy (WTA):

$$\Delta\phi(x) = \arg\max_{\Delta\psi_i, d_j} \left| Q^L(x) + \mathrm{e}^{\mathrm{j}\Delta\psi_i + jk_0 d_j} Q^R(x) \right|^2$$

Yet, alternative decoding mechanisms can be used (e.g., CoG)

DISPARITY TUNING





[adapted from Ohzawa et al, 1997]

PHASE-BASED DISPARITY ESTIMATION

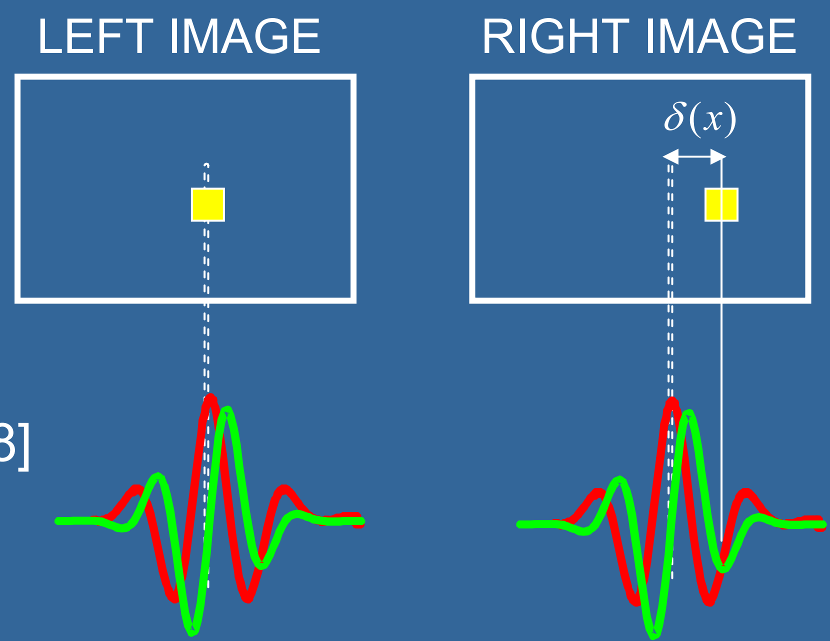
The disparity $\delta(x)$ is defined as the 1D shift necessary to align, along the epipolar lines, the phase values of bandpass filtered versions of the stereo image pair

$$I^L(x) = I^R[x + \delta(x)]$$

$$I^L[x + \delta(x)] \rightarrow \widetilde{I}(k)e^{jk\delta(x)}$$

$$\delta(x) = \frac{\lfloor \phi^L(x) - \phi^R(x) \rfloor_{2\pi}}{k_0}$$

[Sanger, 1988]



$$\lfloor \Delta\phi(x) \rfloor_{2\pi} = \arctan \frac{C^R S^L - C^L S^R}{C^L C^R + S^L S^R}$$

[Solari et al., 2001]

MULTICHANNEL ARCHITECTURE (1)

- Pooling information across spatial scales
  - to remove ambiguities and false matches
  - to cover a wider range of disparity values
- Pooling across orientation channels
  - to “solve” the “aperture problem”
  - to project the displacement respect to the direction of the peak frequency vector on the horizontal epipolar line

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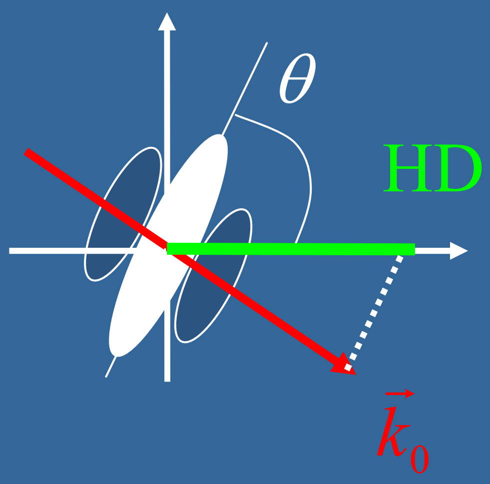
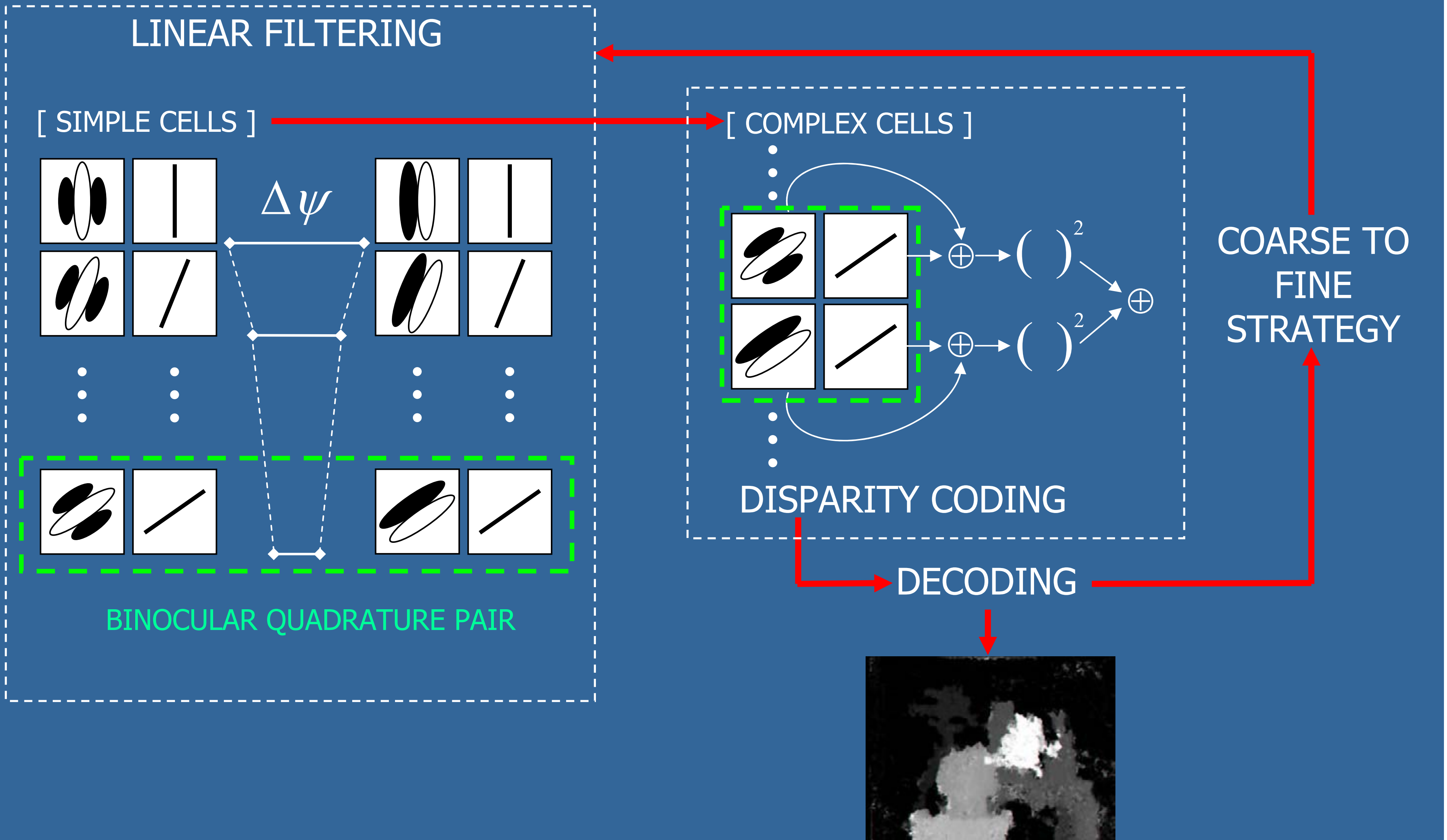


$$\boxed{\mathrm{HD} = \frac{\Delta\phi}{k_0 \sin\theta}}$$

The detectable disparity range becomes

$$-\frac{\pi}{k_0 \sin\theta} < \mathrm{HD} < \frac{\pi}{k_0 \sin\theta}$$

MULTICHANNEL ARCHITECTURE (2)



TOWARDS A GENERALIZED ARCHITECTURE

- Handling horizontal and vertical disparities
- Vergence in the loop:

refinement of vergence

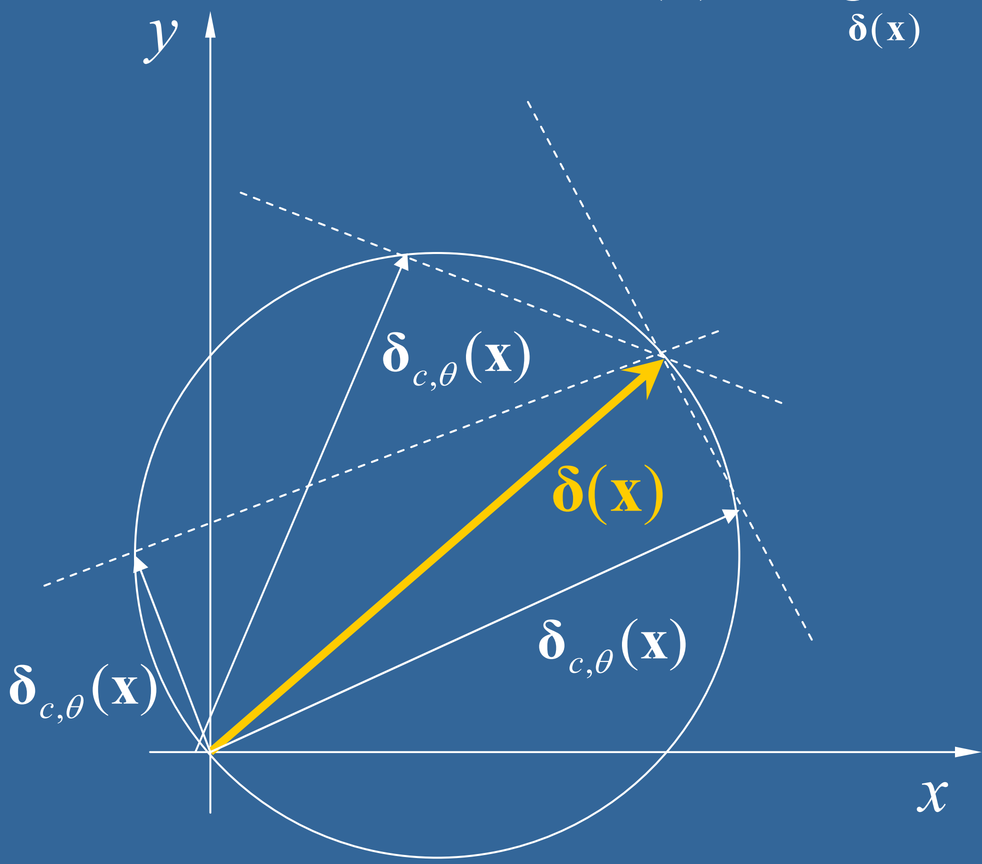
→ vs.

refinement of the disparity map

Take care that: mechanisms guiding vergence eye movements are different from those supporting depth perception!

HANDLING HD+VD (VECTOR DISPARITY)

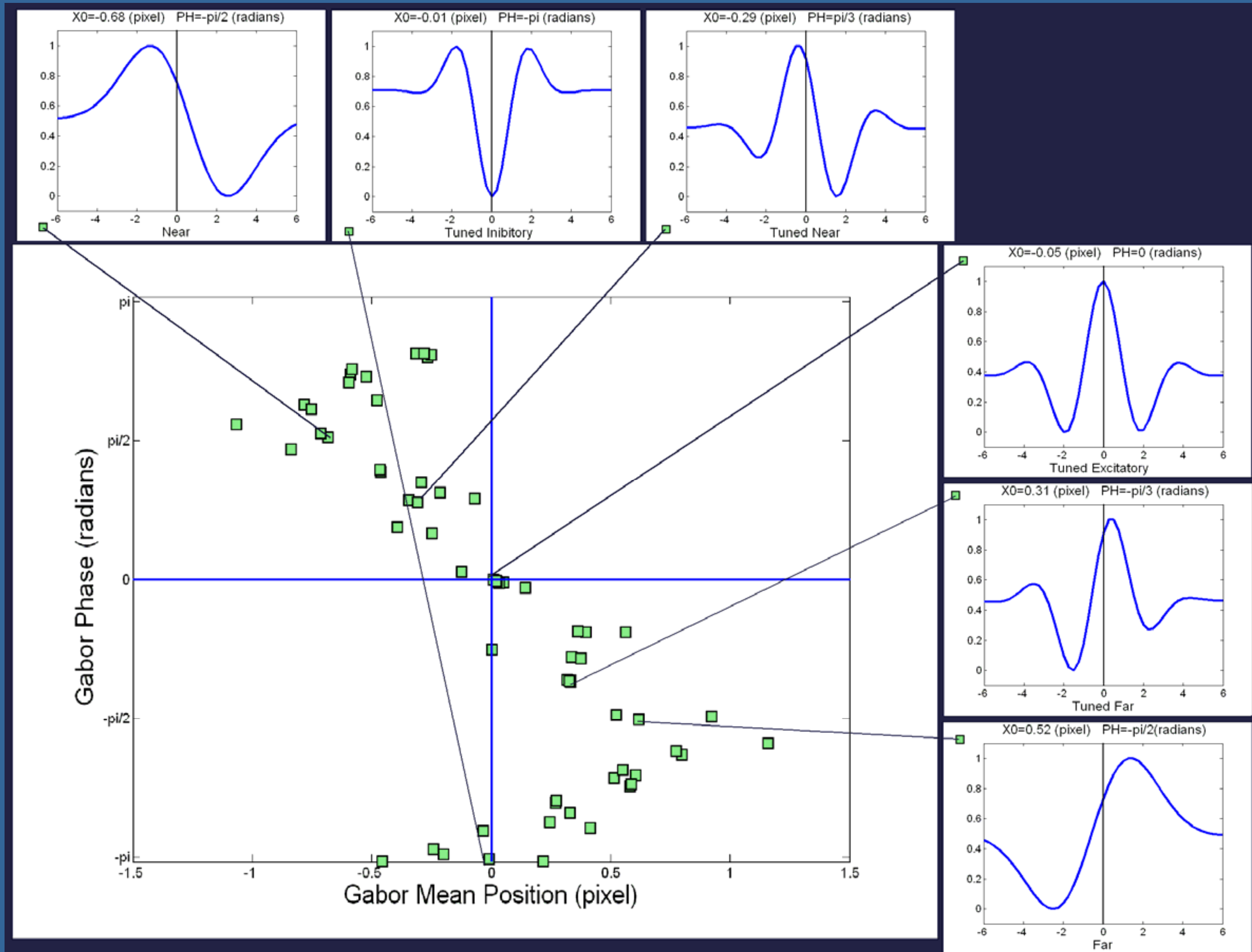
$$\boldsymbol{\delta}^*(\mathbf{x}) = \arg\min_{\boldsymbol{\delta}(\mathbf{x})} \sum_{\theta} \left(\left\| \boldsymbol{\delta}_{c,\theta}(\mathbf{x}) \right\| - \boldsymbol{\delta}(\mathbf{x})^T \frac{\boldsymbol{\delta}_{c,\theta}(\mathbf{x})}{\left\| \boldsymbol{\delta}_{c,\theta}(\mathbf{x}) \right\|} \right)^2$$



[cf. Theimer & Mallot, 1994]

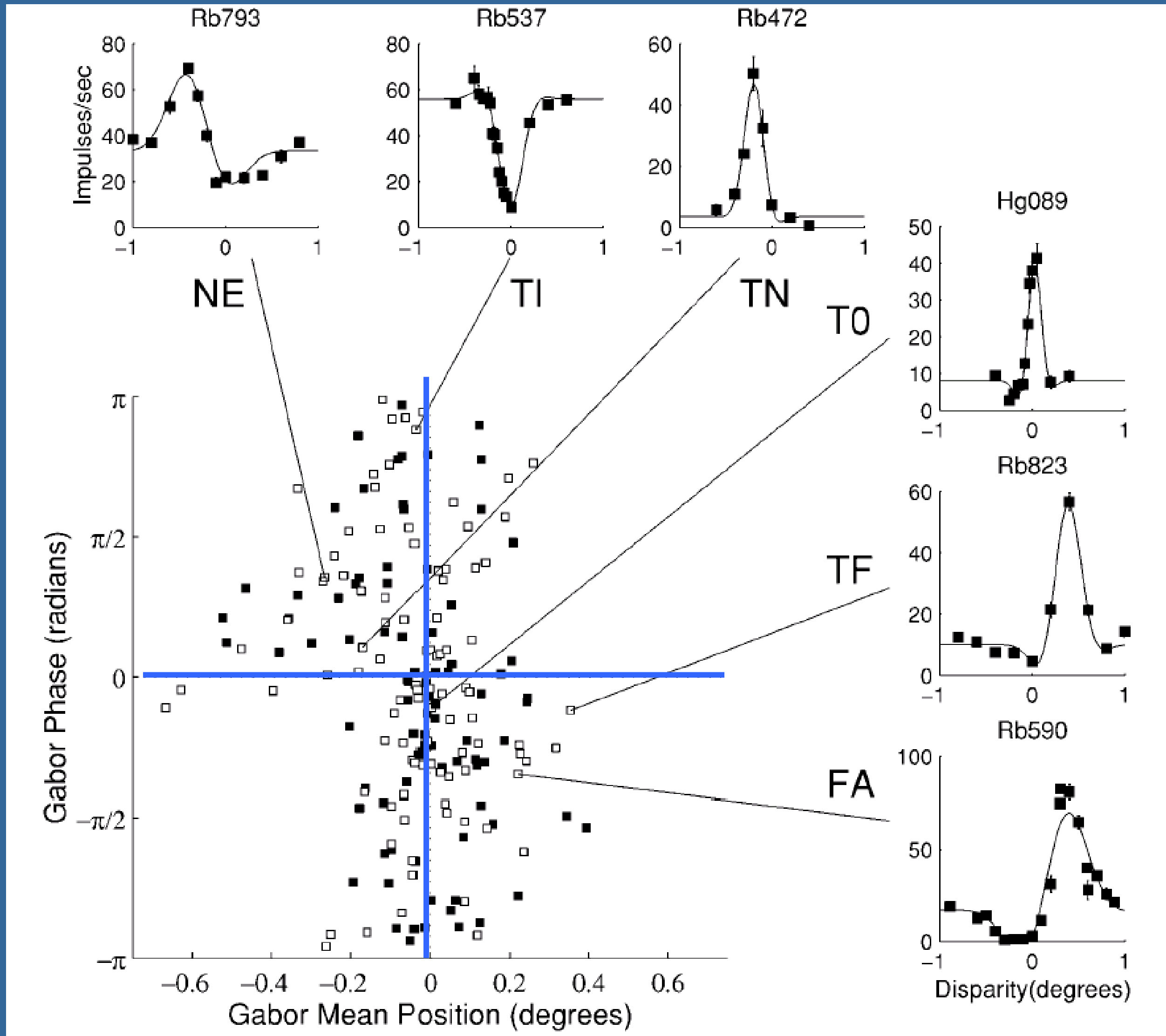
Disparity is defined as the vector difference in positions of identified corresponding points in the left and right eyes, each measured with respect to the point of fixation as origin.

MODEL'S DISPARITY TUNING CURVES



REAL DISPARITY TUNING CURVES

[Prince et al., 2002]



JOINT CODING (metamerism)

- Quantitative models of early cortical processing start with a linear convolution of the image with a RF profile

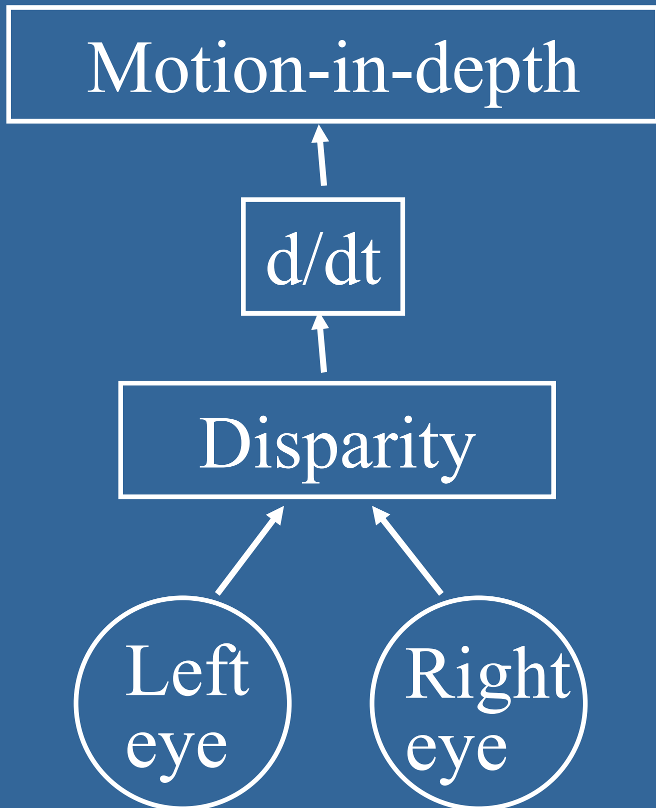
→ They do not encode features independently!

Several stimulus parameters (position, orientation, spatial frequency, motion) are encoded simultaneously

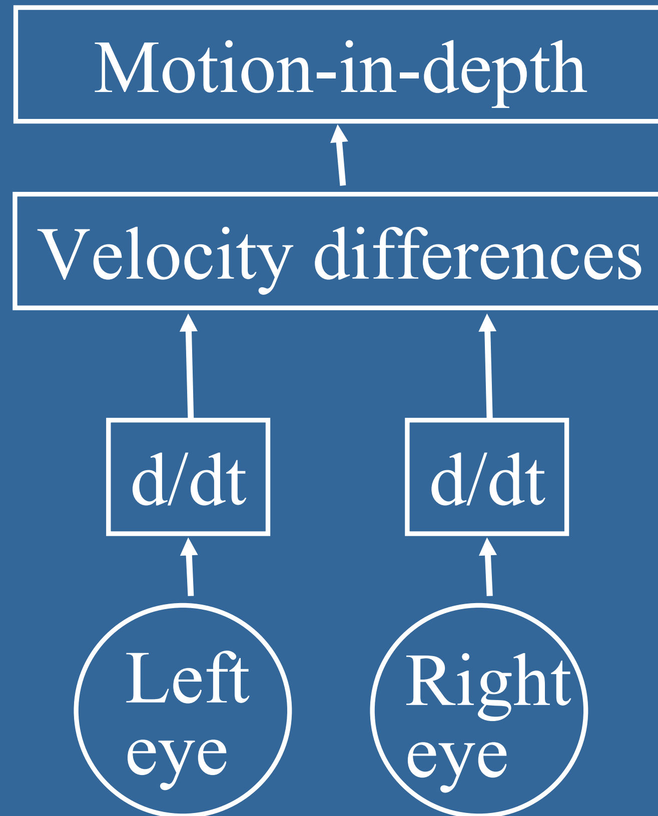
- How to encode 2nd-order disparities independently of 1st-order ones?

MID as a case study

COMPUTATIONAL THEORY (1)



Disparity rate of change



Interocular velocity difference

COMPUTATIONAL THEORY (2)

Motion-in-depth

The total rate of variation of disparity can be written as

$$\frac{d\delta}{dt} = \frac{\partial\delta}{\partial t} + \frac{v^L}{k_0}(\phi_x^L - \phi_x^R)$$

Considering the conservation properties of local phase

Thus $$\phi_x^L = -\frac{\phi_t^L}{v^L} \quad \text{and} \quad \phi_x^R = -\frac{\phi_t^R}{v^R}$$

Information hold in the interocular velocity difference is the same of that derived from the derivative of binocular disparity, if a phase-based disparity encoding scheme is assumed

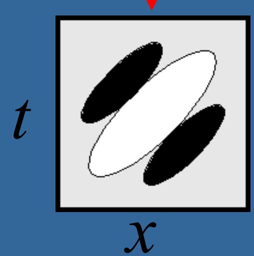
NEUROMORPHIC IMPLEMENTATION

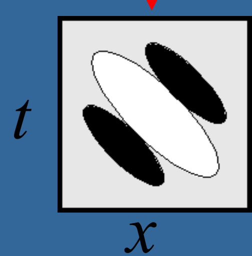
Partial derivative of disparity can be expressed through convolutions with a set of spatiotemporal filters whose shapes resemble simple cell RFs

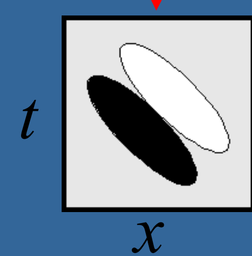
$$\frac{\partial \delta}{\partial t} = \left[\frac{S_t^L C^L - S^L C_t^L}{(S^L)^2 + (C^L)^2} - \frac{S_t^R C^R - S^R C_t^R}{(S^R)^2 + (C^R)^2} \right] \frac{1}{k_0}$$

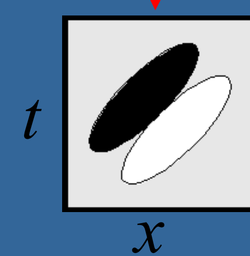
where

$$4SC_t = (S + C_t)^2 - (C_t - S)^2, \qquad 4S_tC = (S_t + C)^2 - (S_t - C)^2$$

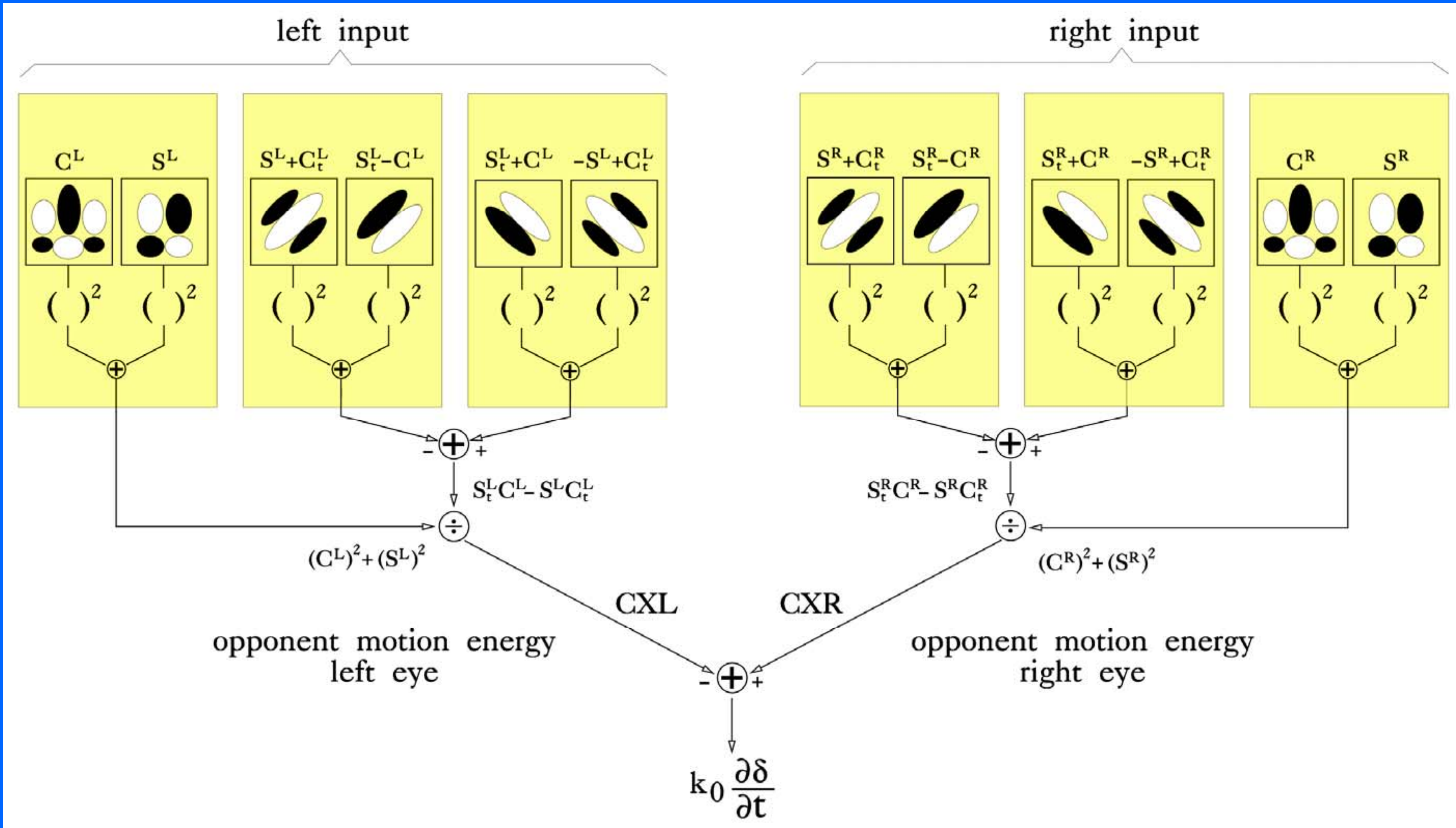








NEUROMORPHIC IMPLEMENTATION



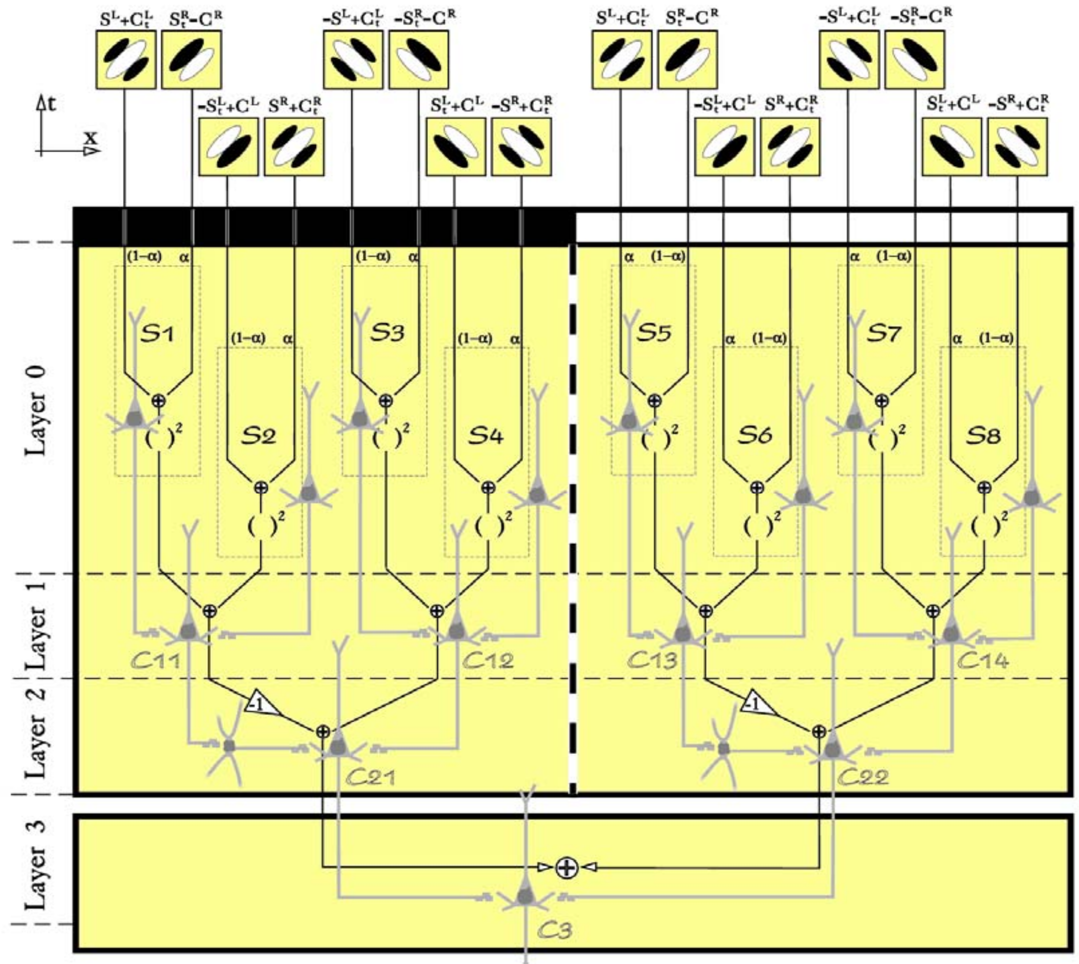
NEUROMORPHIC IMPLEMENTATION

$$S_1 = (1-\alpha)(C_t^L + S^L) - \alpha(C^R - S_t^R)$$
$$S_2 = (1-\alpha)(C^L + S_t^L) + \alpha(C_t^R + S^R)$$
$$S_3 = (1-\alpha)(C_t^L - S^L) - \alpha(C^R + S_t^R)$$
$$S_4 = (1-\alpha)(C^L + S_t^L) + \alpha(C_t^R - S^R)$$
$$S_5 = \alpha(C_t^L + S^L) - (1-\alpha)(C^R - S_t^R)$$
$$S_6 = \alpha(C^L - S_t^L) + (1-\alpha)(C_t^R + S^R)$$
$$S_7 = \alpha(C_t^L - S^L) - (1-\alpha)(C^R + S_t^R)$$
$$S_8 = \alpha(C^L + S_t^L) + (1-\alpha)(C_t^R - S^R)$$

$$C_{11} = S_1^2 + S_2^2 \qquad C_{12} = S_3^2 + S_4^2$$
$$C_{13} = S_5^2 + S_6^2 \qquad C_{14} = S_7^2 + S_8^2$$

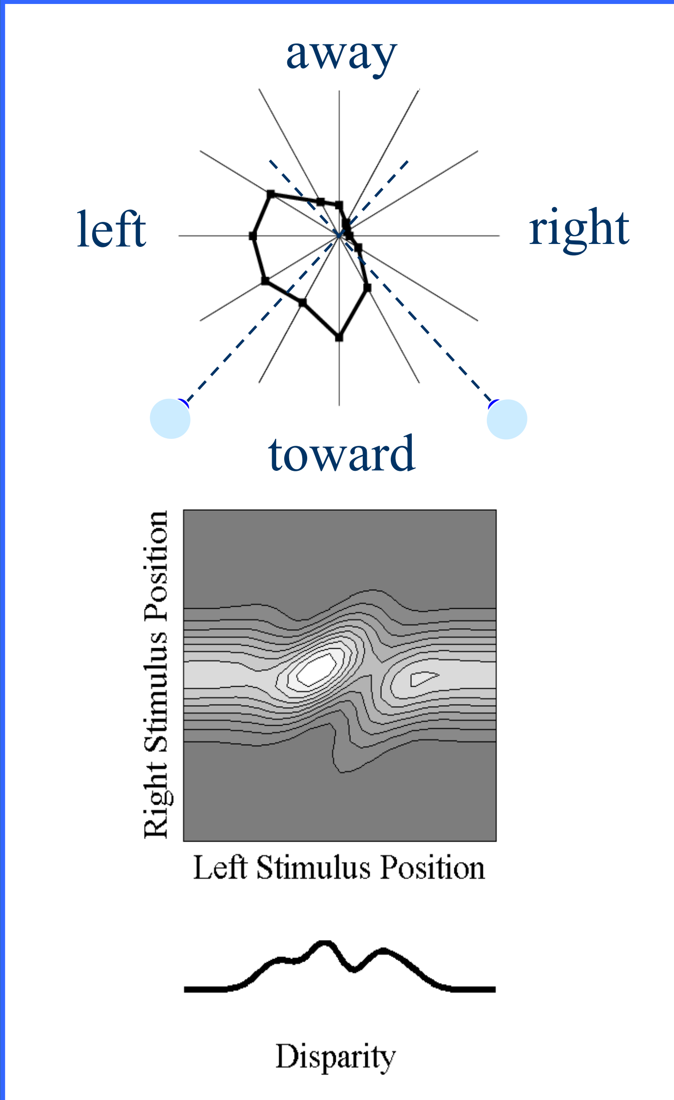
$$C_{21} = C_{12} - C_{11}$$
$$C_{22} = C_{13} - C_{14}$$

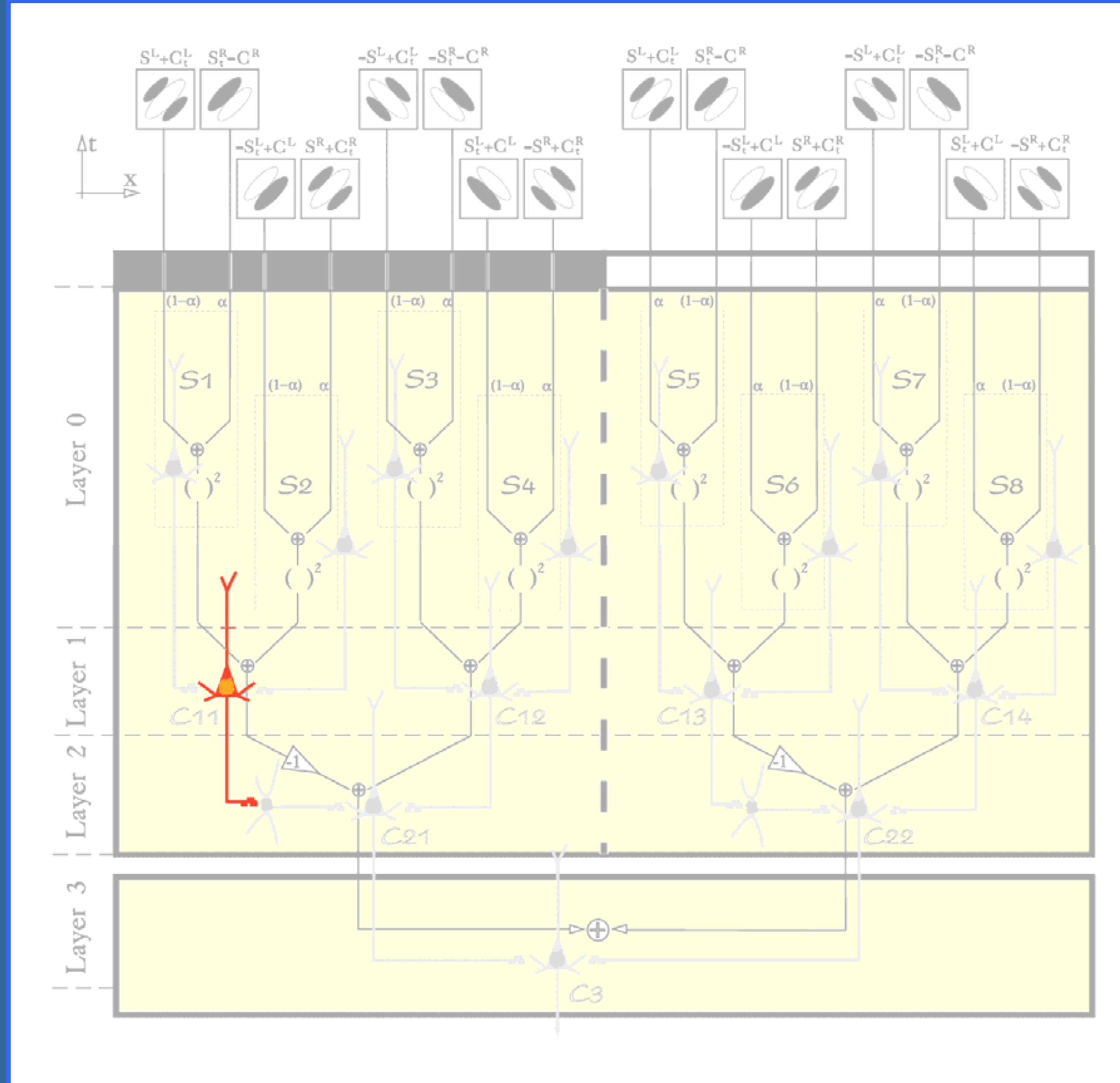
$$C_3 = C_{21} + C_{22}$$



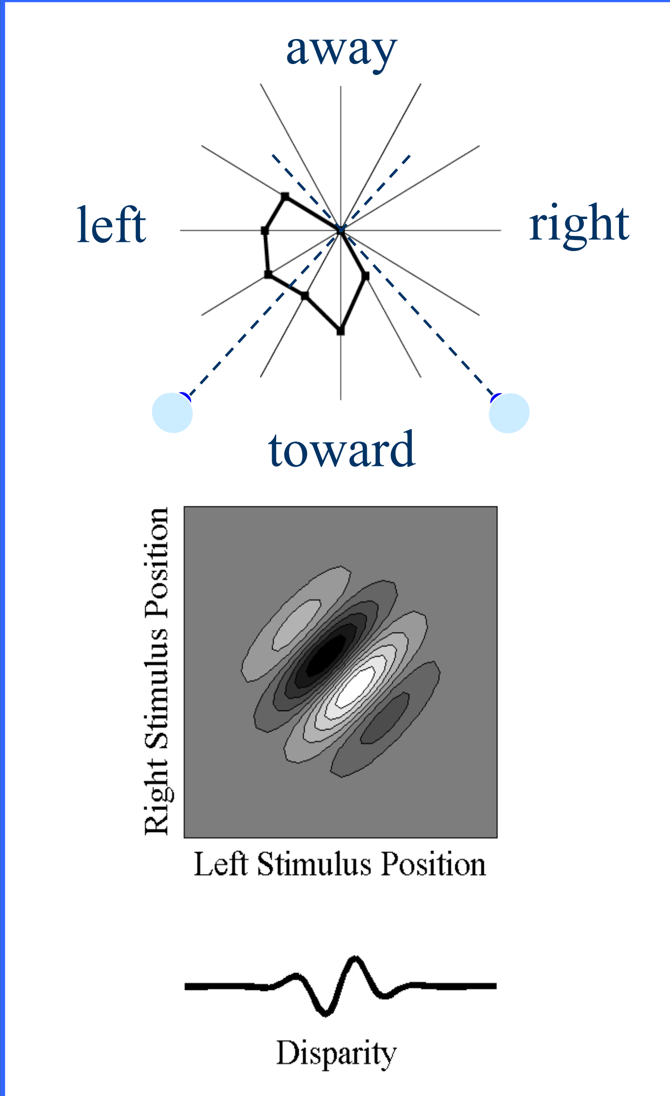
$$(1-2\alpha)(S_t^L C^L - S^L C_t^L - S_t^R C^R + S^R C_t^R$$

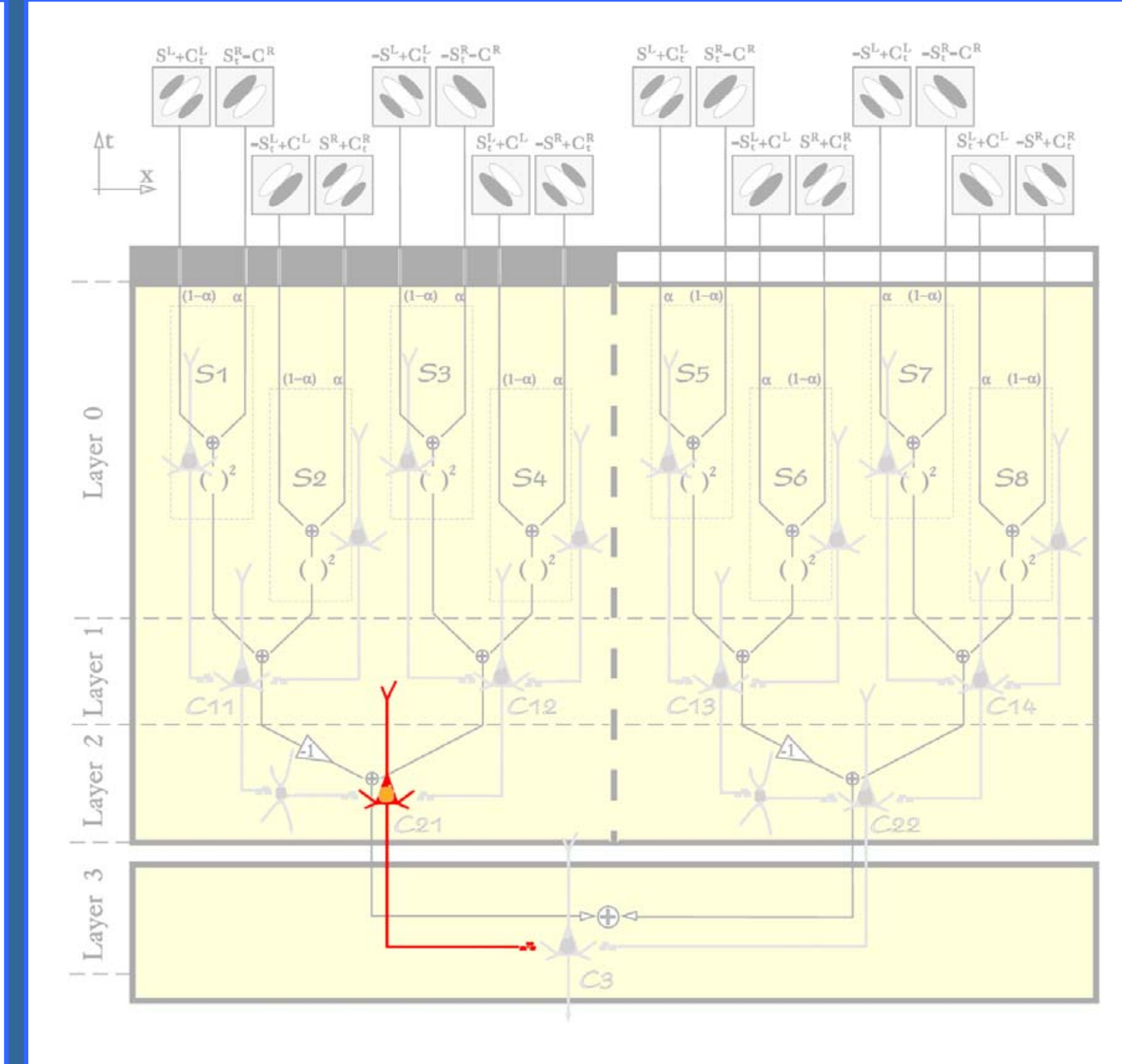
NEUROMORPHIC IMPLEMENTATION



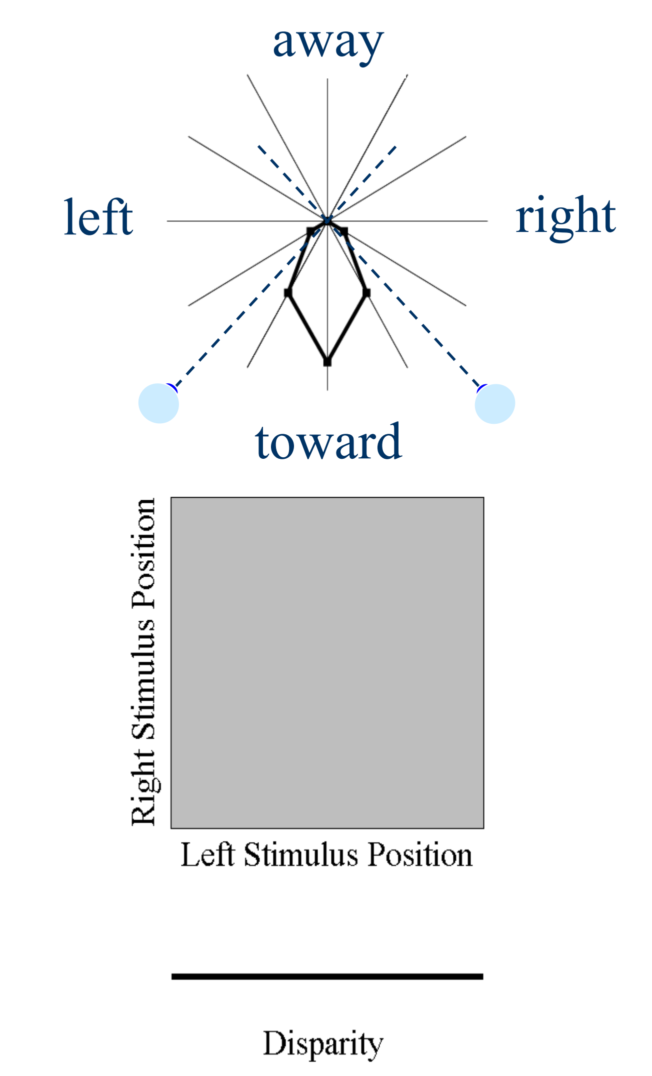


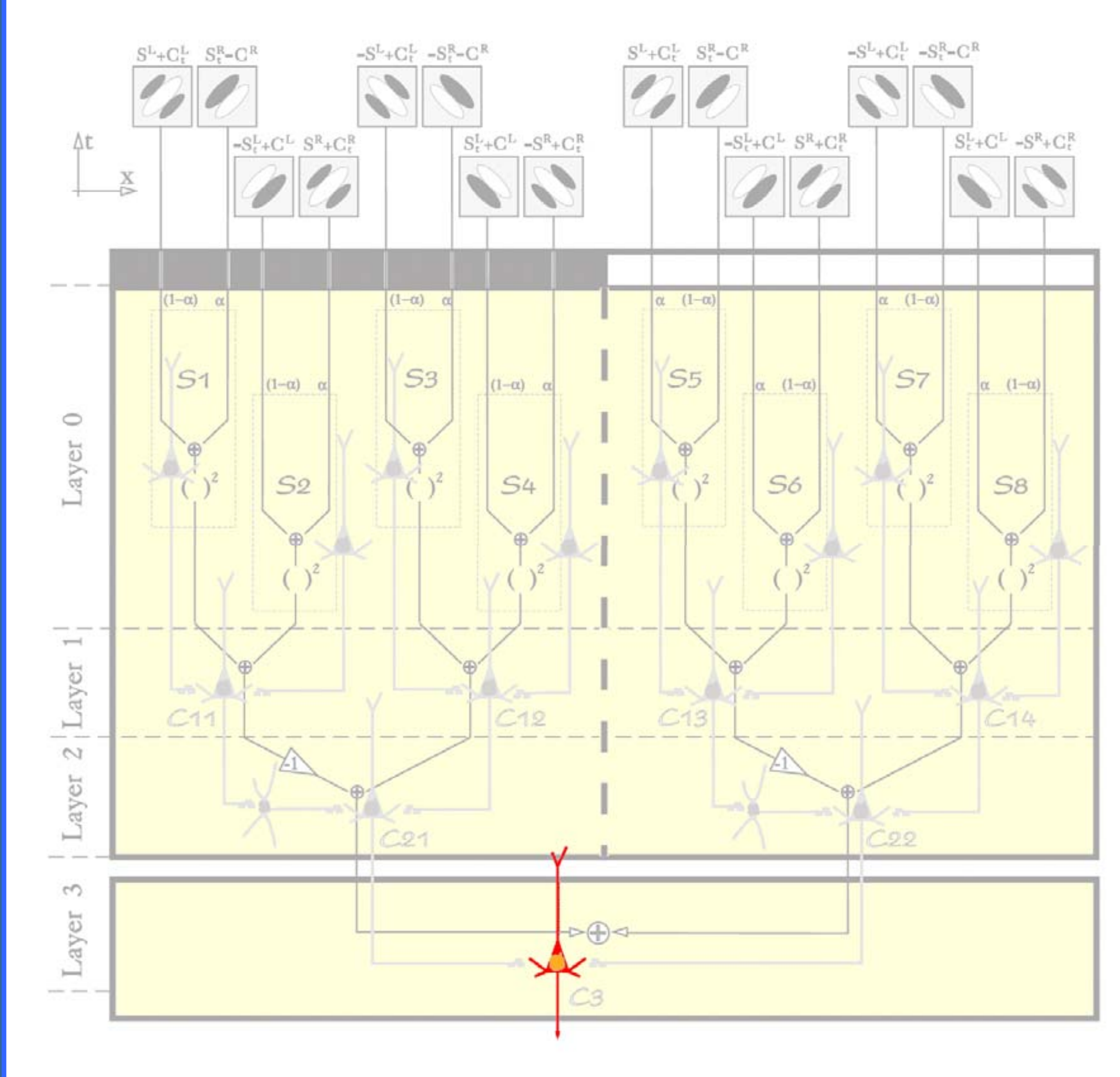
NEUROMORPHIC IMPLEMENTATION





NEUROMORPHIC IMPLEMENTATION





MODEL'S PREDICTIONS

MONO ocular dominance space **BIN0**

motion energy models

0 , 1 0.5

binocular energy models

Adelson & Bergen, 1985

Emerson et *al.*, 1992

Continuous responses to *mixed* motion and disparity components

Ohzawa et al., 91, 97, 99

Qian et al., 95, 97, 2000

Fleet et al., 1996

Cell type 1: disparity BUT NOT motion-in-depth tuning

Cell type 2: motion-in-depth BUT NOT disparity tuning

CONCLUSIONS

The proposed architecture

- represents a structural way to obtain dynamic disparity descriptors of growing complexity:

  <u>Simple cells</u> → provide a general (harmonic) representation on which to base a variety of visual functions

  <u>Complex cells</u> → by proper combinations of the simple cells' outputs become specifically tuned to different features

- combines connectionism paradigms with signal processing

- provides "entry points" for adaptation and learning

THE EYESHOTS' PERSPECTIVE

Approach: active measures in the plenoptic space

Issues to be addressed:

- Active stereopsis [WP2]

  Static disparity

  - V1 S-cell network for 1st-order (positional) disparity coding ✓
  - Space-variant sampling of retinal space (log-polar)
  - Systematic multilayer & multidimensional extension of the "energy circuits" for 2nd-order disparity coding

  Improve stereo matching

  Dynamic disparity

  - Direct use of MID information for guiding active scrutiny

- Depth judgements [WP3]
  - Hierarchical extension of the architecture to handle relative disparity

- Oculomotor issues [WP1]

THE EYESHOTS' PERSPECTIVE

Approach: active measures in the plenoptic space

Issues to be addressed:

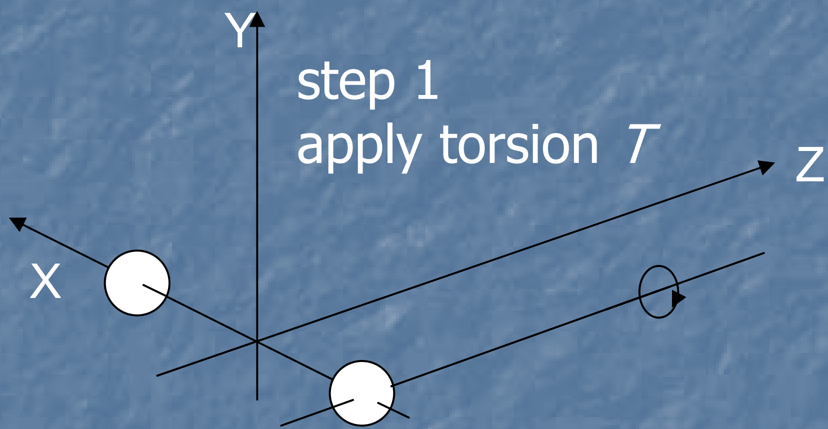
- Active stereopsis [WP2]
  - Static disparity
    - V1 S-cell network for 1st-order (positional) disparity coding ✓
    - Space-variant sampling of retinal space (log-polar)
    - Systematic multilayer & multidimensional extension of the "energy circuits" for 2nd-order disparity coding
  - Dynamic disparity
    - Direct use of MID information for guiding active scrutiny
- Depth judgements [WP3]
  - Hierarchical extension of the architecture to handle relative disparity
- Oculomotor issues [WP5]

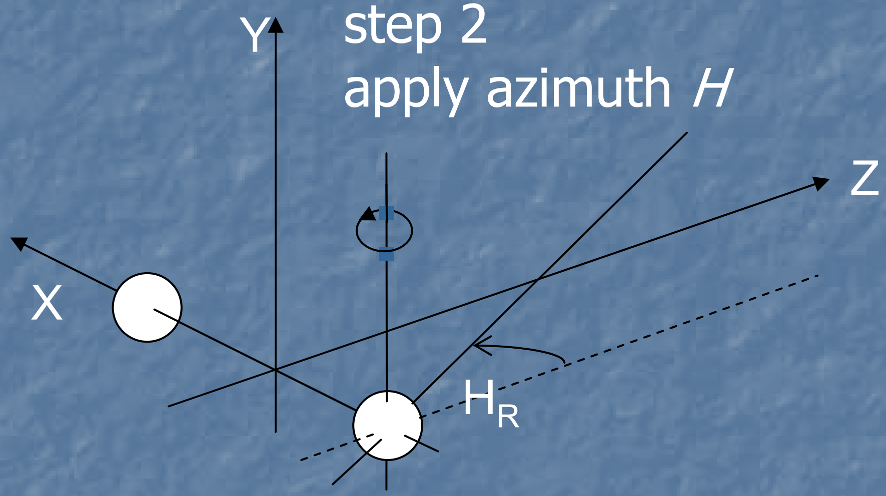
Gain fields

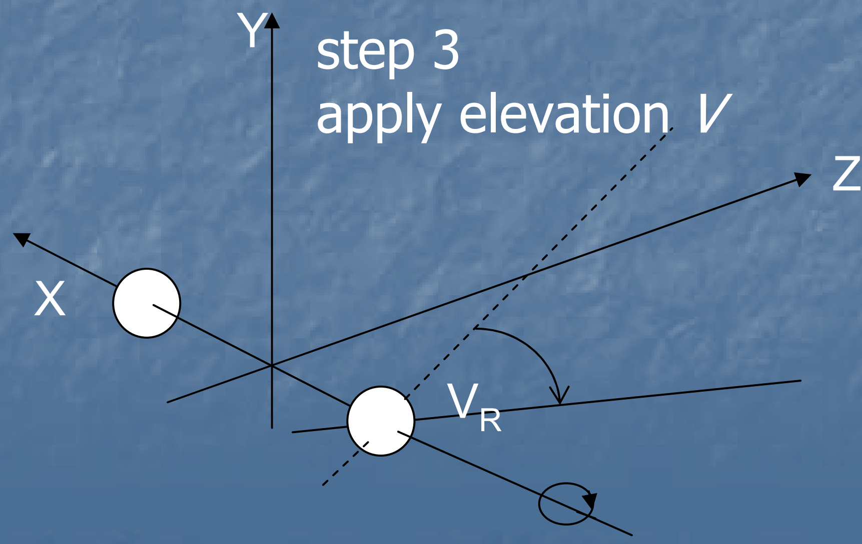
ACTIVE STEREOPSIS AND EYE MOVEMENTS

Andrea Canessa

Helmhotz coordinates







Quaternions

- Many papers in the eye-movement literature specify eye positions by quaternions.
- A rotation through an angle ε about a unit vector **n** can be represented by the quaternion

$$q = \left(\cos\frac{\varepsilon}{2}\right) + \left(\sin\frac{\varepsilon}{2}\right)\mathbf{n} = q_0 + \mathbf{q}$$

Helmhotz rotations

- The three rotations which move the eye from its reference position into a position with Helmholtz coordinates (H,V,T), are a rotation through T about **Z**, a rotation through H about **Y** and a rotation through V about **X**.

$$q_T = \left(\cos\frac{T}{2}\right) + \left(\sin\frac{T}{2}\right)\mathbf{Z} \qquad q_H = \left(\cos\frac{H}{2}\right) + \left(\sin\frac{H}{2}\right)\mathbf{Y}$$

$$q_V = \left(\cos\frac{V}{2}\right) + \left(\sin\frac{V}{2}\right)\mathbf{X}$$

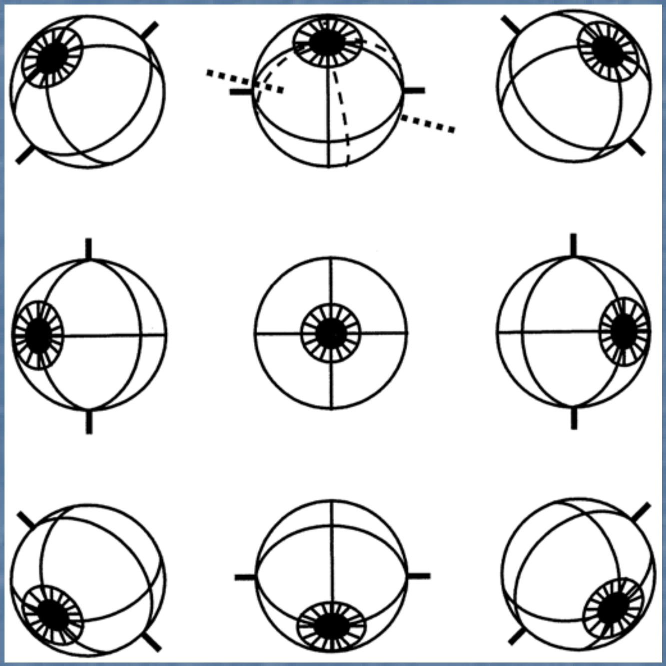
Helmhotz rotations

- So, multiplying these three together in the order $q_V q_H q_T$ gives the quaternion

$$
\begin{aligned}
q \;=\; & \left(\cos\frac{V}{2}\cos\frac{H}{2}\cos\frac{T}{2}-\sin\frac{V}{2}\sin\frac{H}{2}\sin\frac{T}{2}\right) \\
& \left(\cos\frac{V}{2}\sin\frac{H}{2}\sin\frac{T}{2}+\sin\frac{V}{2}\cos\frac{H}{2}\cos\frac{T}{2}\right)\mathbf{X} \\
& +\left(\cos\frac{V}{2}\sin\frac{H}{2}\cos\frac{T}{2}-\sin\frac{V}{2}\cos\frac{H}{2}\sin\frac{T}{2}\right)\mathbf{Y} \\
& +\left(\cos\frac{V}{2}\cos\frac{H}{2}\sin\frac{T}{2}+\sin\frac{V}{2}\sin\frac{H}{2}\cos\frac{T}{2}\right)\mathbf{Z}
\end{aligned}
$$

Listing’s Law

- Listing’s law states that, when the head is fixed, there is an eye position called primary position, such that the eye assumes only those orientations that can be reached from primary position by a single rotation about an axis in a plane called Listing’s plane.

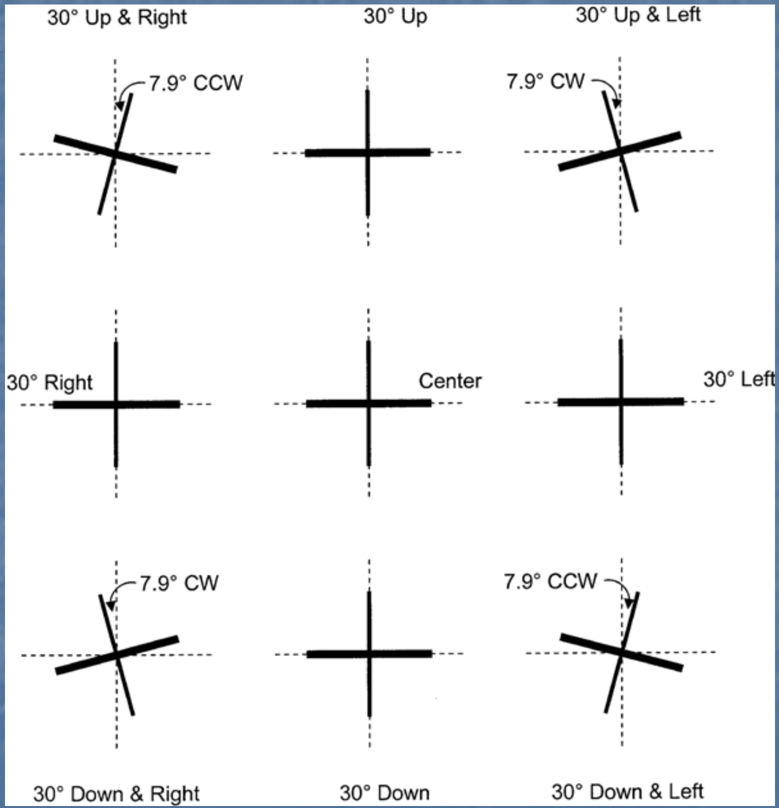


- This plane is orthogonal to the line of sight when the eye is in primary position.

Listing’s Law

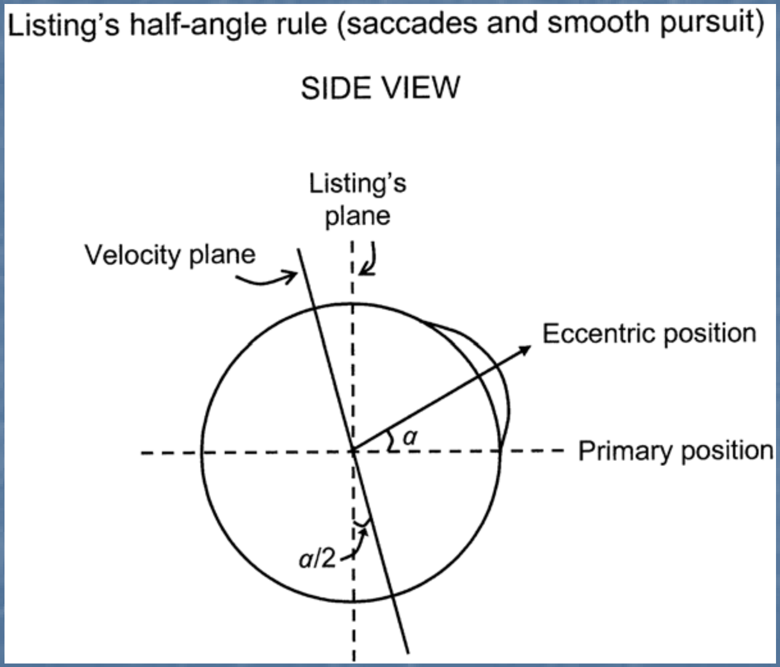
- Listing's law requires that $q_T = 0$
- In Helmholtz coordinates, Listing’s law says:

$$T \cong -\frac{HV}{2}$$



Listing’s half - angle rule

- What if the eye starts its rotation from an eccentric eye position?
- The eye rotates about axes that lie in a plane (velocity plane) but this plane is tilted in the same direction as the line of sight but only half as much



What about a binocular system?

- If the eyes continued to obey Listing's law when they converge the torsional angles differ:

$$T_r \cong -\frac{H_r V}{2} \qquad T_l \cong -\frac{H_l V}{2}$$

- This leads to nonzero torsional disparity

$$T_{disp} = T_r - T_l \cong -\frac{\nu V}{2}$$

where ν is the vergence angle

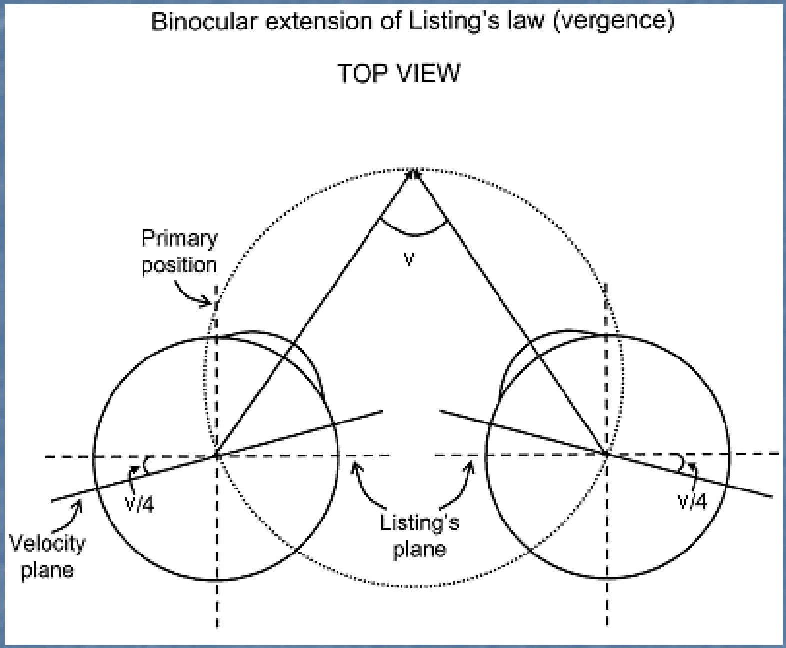
Extending Listing's law: L2

- We could avoid this disparity by breaking Listing's law, adding to each eye some extra torsion that varies in the appropriate way with ν and V:

$$T_r = -\frac{H_r V}{2} + \frac{\nu V}{4} = -\frac{\gamma V}{2}$$

$$T_l = -\frac{H_l V}{2} - \frac{\nu V}{4} = -\frac{\gamma V}{2}$$

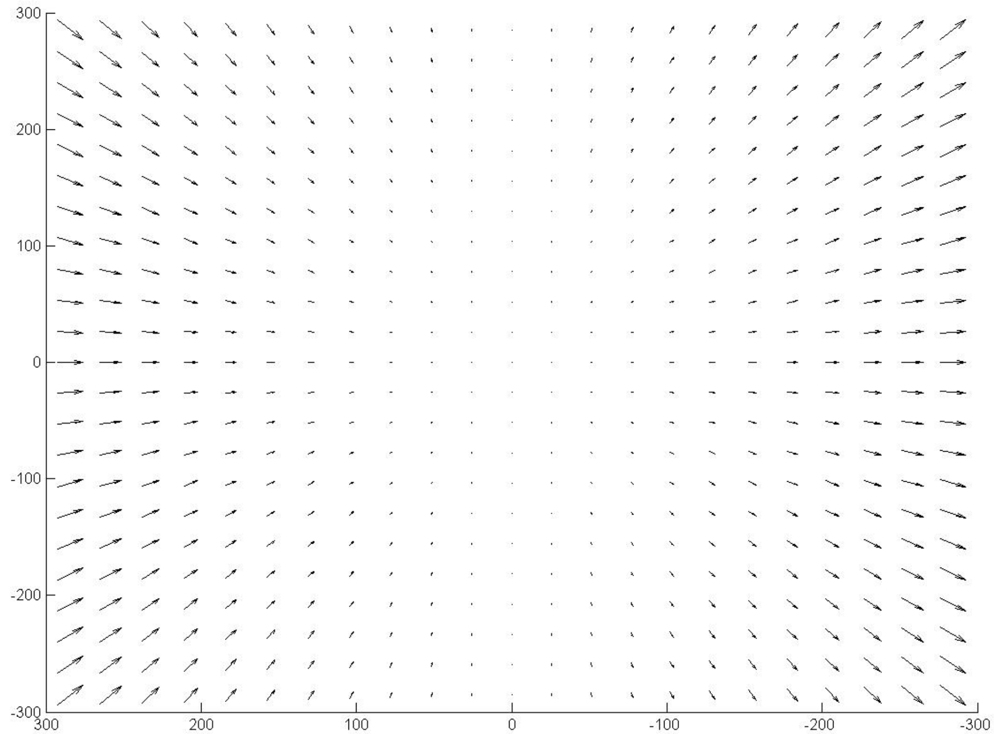
where γ is the version angle



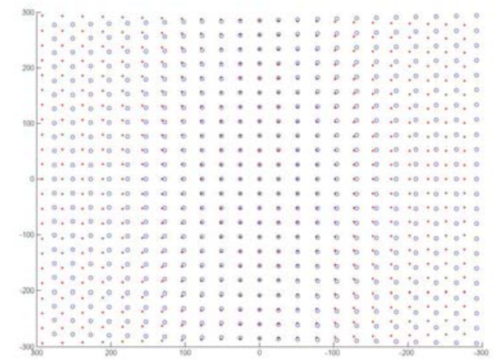
Primary position

Gaze: 0°, 0°   Distance: 30cm

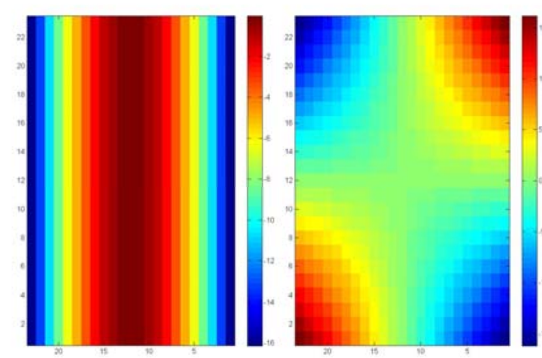
Plane: FrontoParallel



Disparity flow



Retinal projection

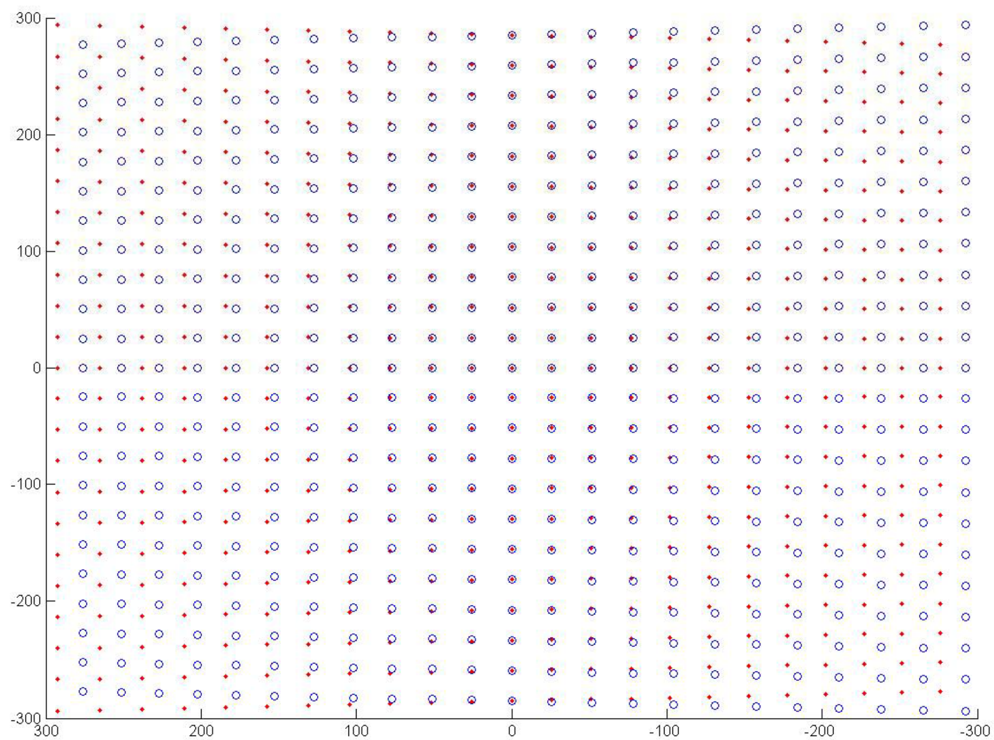


Horizontal Disparity   Vertical Disparity

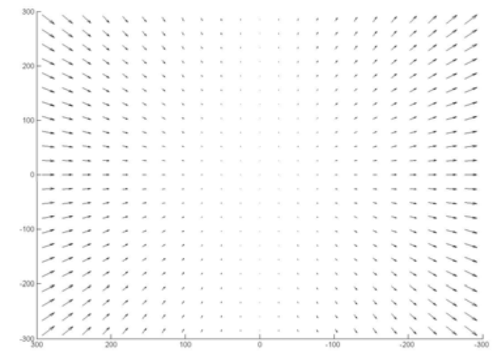
Primary position

Gaze: 0°, 0°   Distance: 30cm

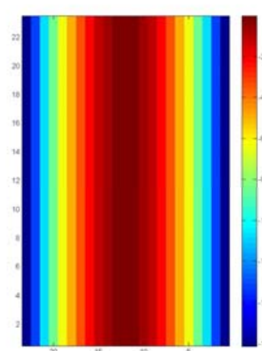
Plane: FrontoParallel



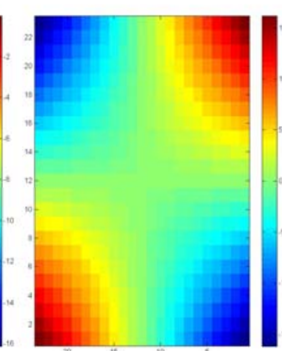
Retinal projection



Disparity flow



Horizontal Disparity

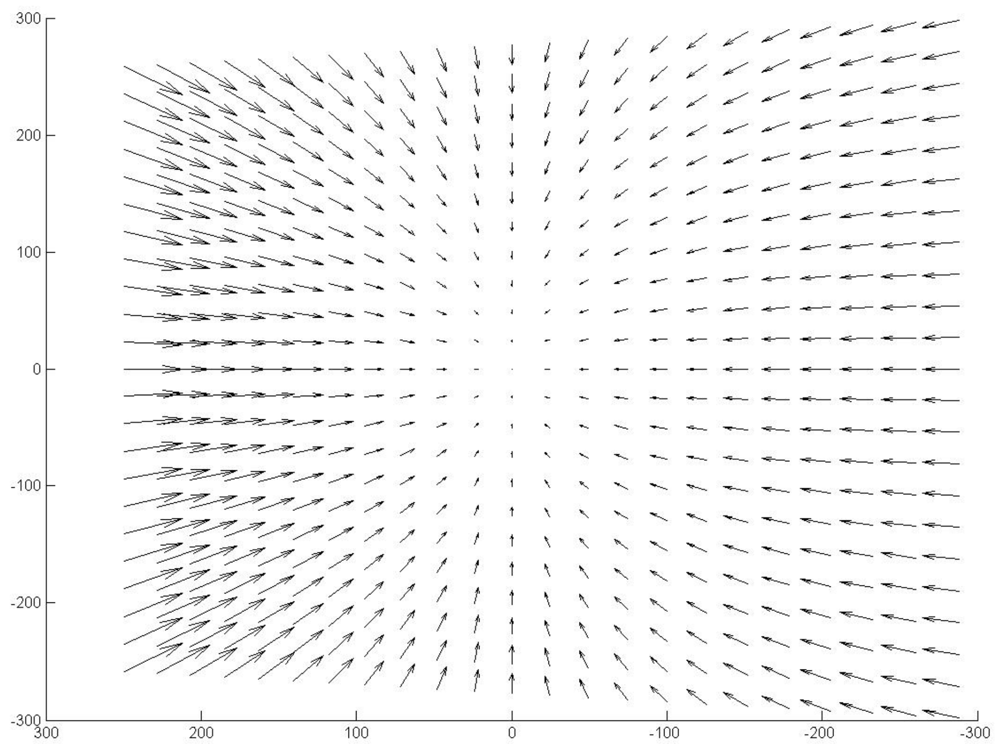


Vertical Disparity

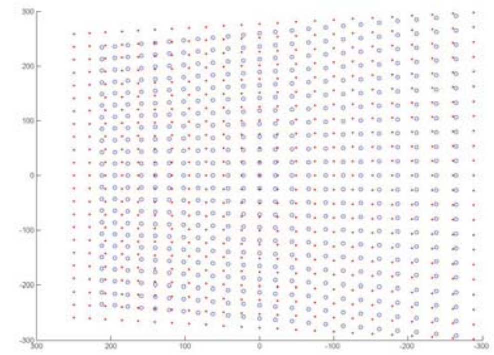
Secondary position

Gaze: 20°, 0°   Distance: 30cm

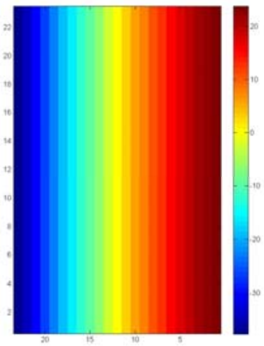
Plane: FrontoParallel



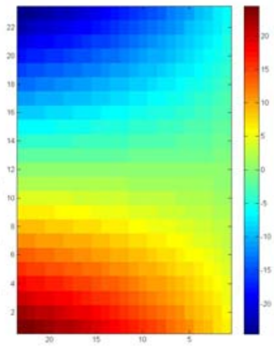
Disparity flow



Retinal projection



Horizontal Disparity

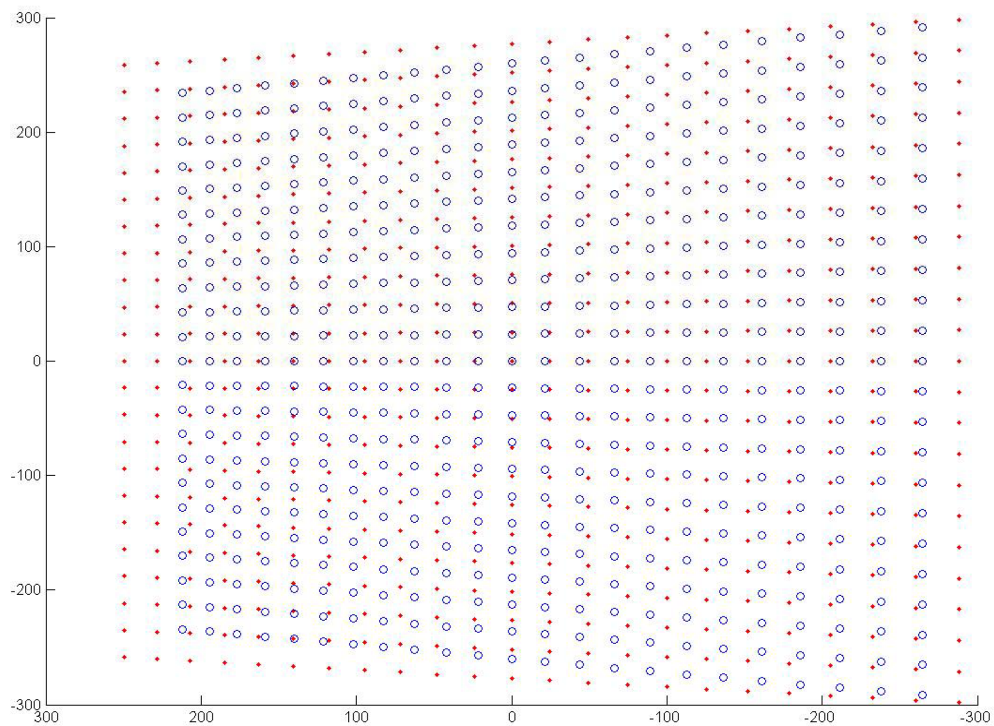


Vertical Disparity

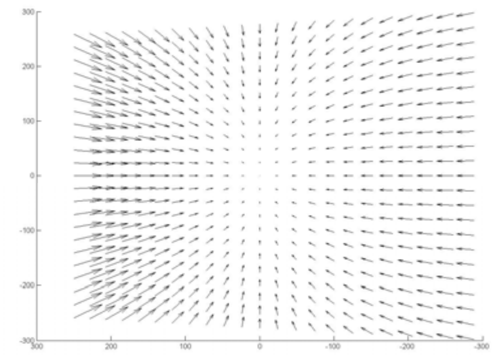
Secondary position

Gaze: 20°, 0° Distance: 30cm

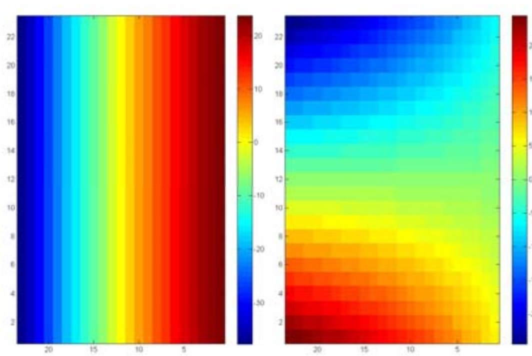
Plane: FrontoParallel



Retinal projection



Disparity flow



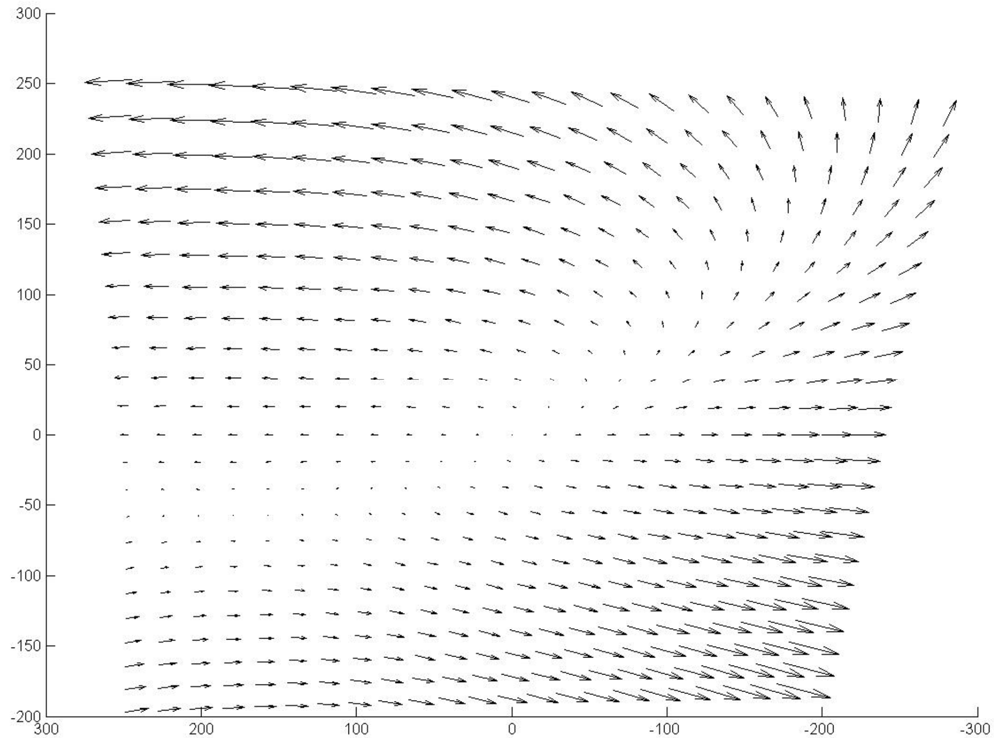
Horizontal Disparity

Vertical Disparity

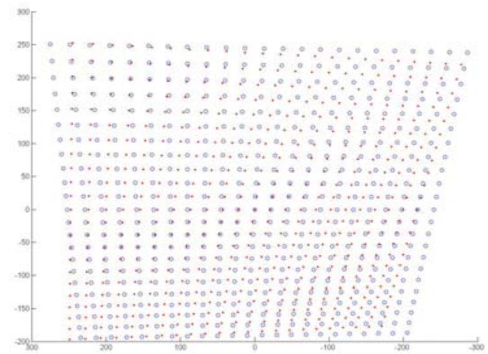
Tertiary position

Gaze: -10°, 30° Distance: 30cm

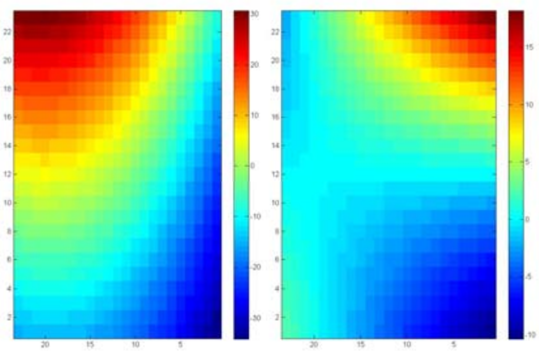
Plane: Frontoparallel



Disparity flow



Retinal projection



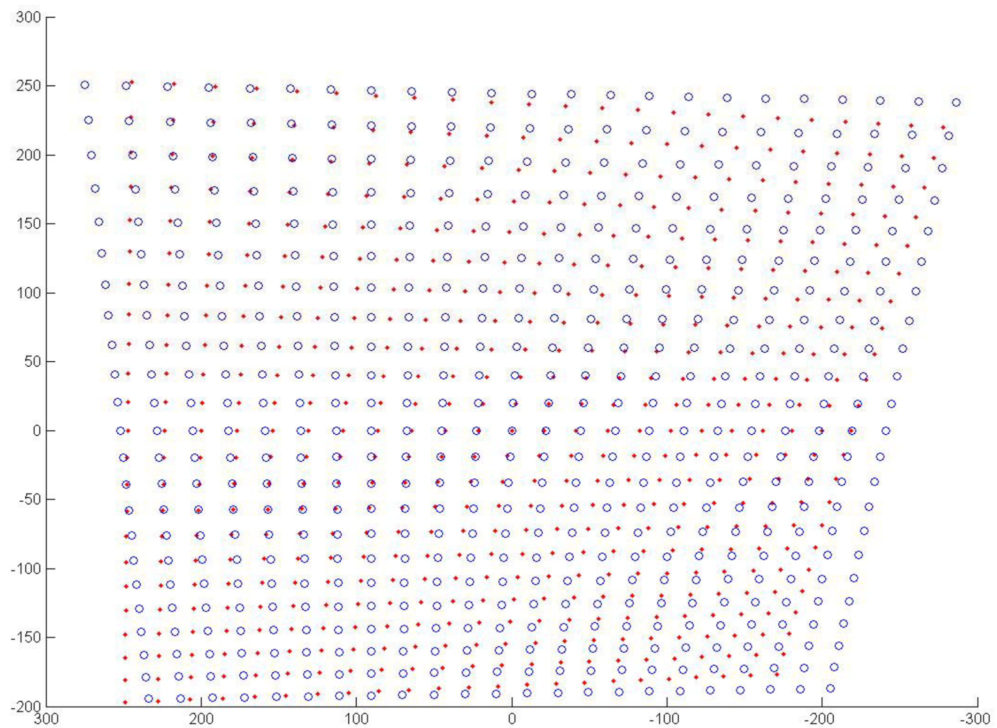
Horizontal Disparity

Vertical Disparity

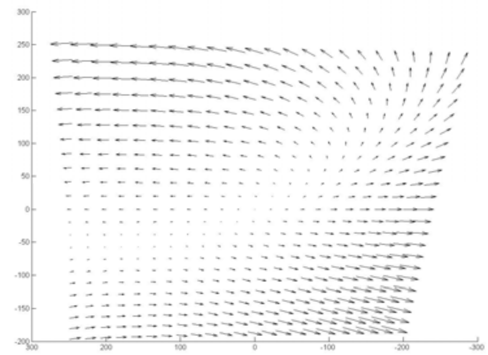
Tertiary position

Gaze: -10°, 30° Distance: 30cm

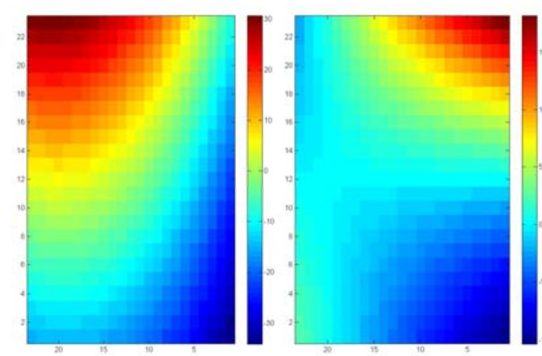
Plane: Frontoparallel



Retinal projection



Disparity flow

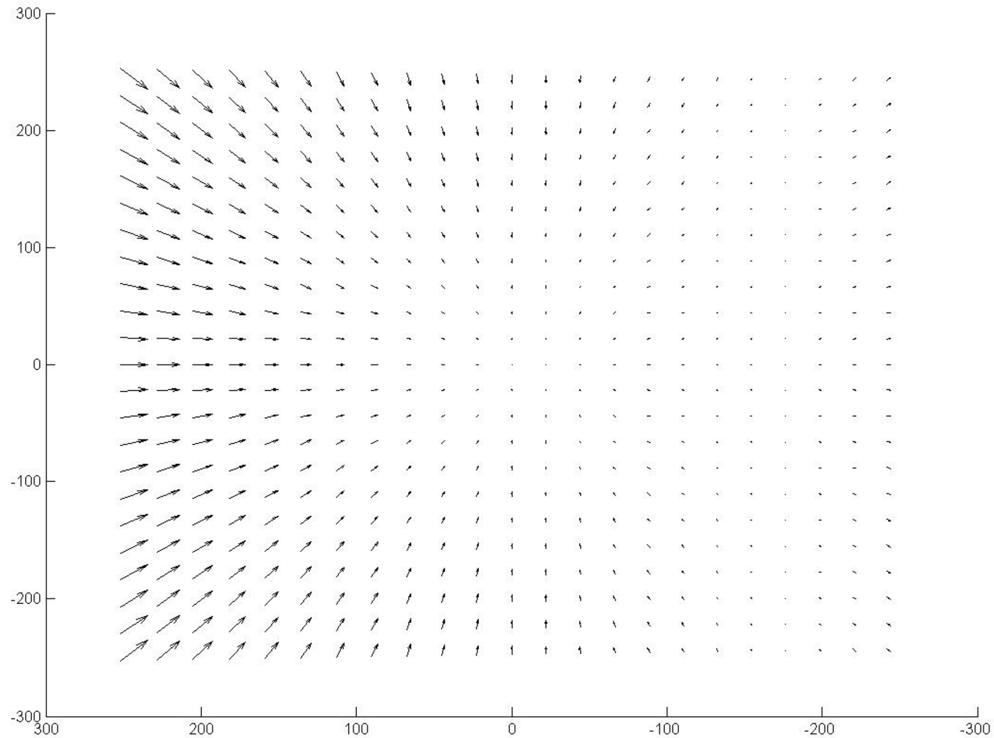


Horizontal Disparity   Vertical Disparity

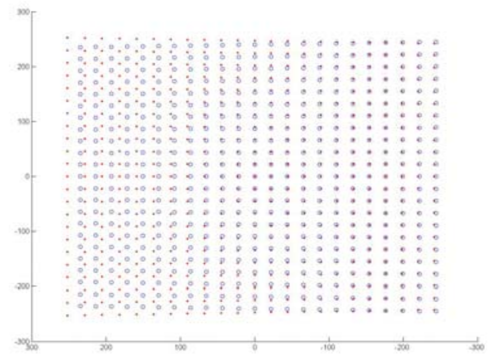
No torsion

Gaze: 10°, 30°   Distance: 30cm

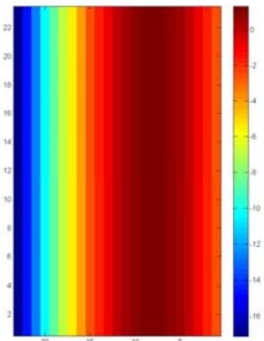
Plane: Perpendicular sight line



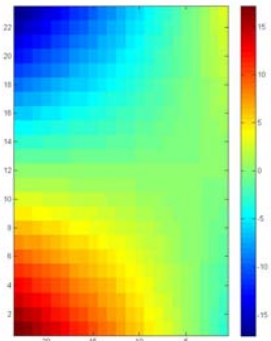
Disparity flow



Retinal projection



Horizontal Disparity

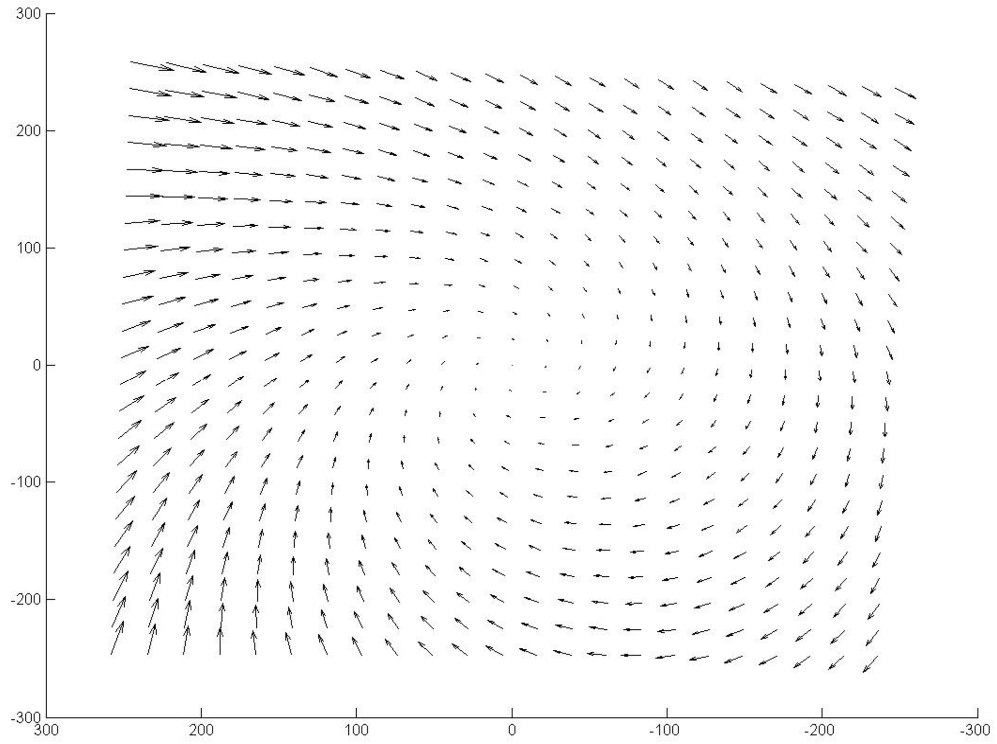


Vertical Disparity

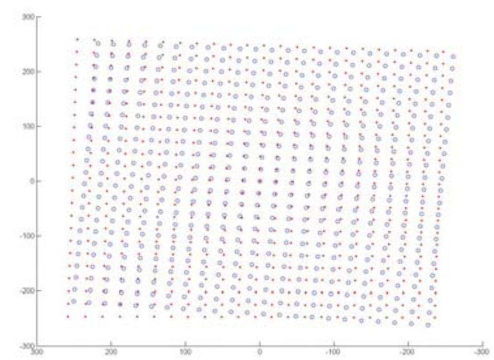
Listing's law

Gaze: 10°, 30°   Distance: 30cm

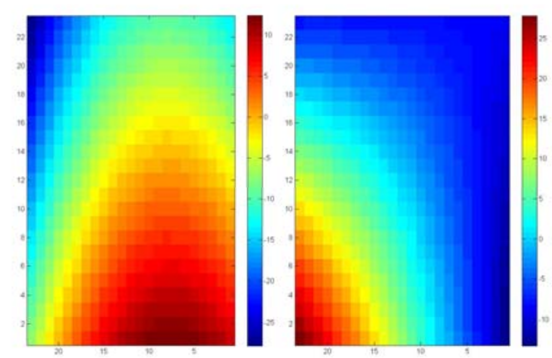
Plane: Perpendicular sight line



Disparity flow



Retinal projection

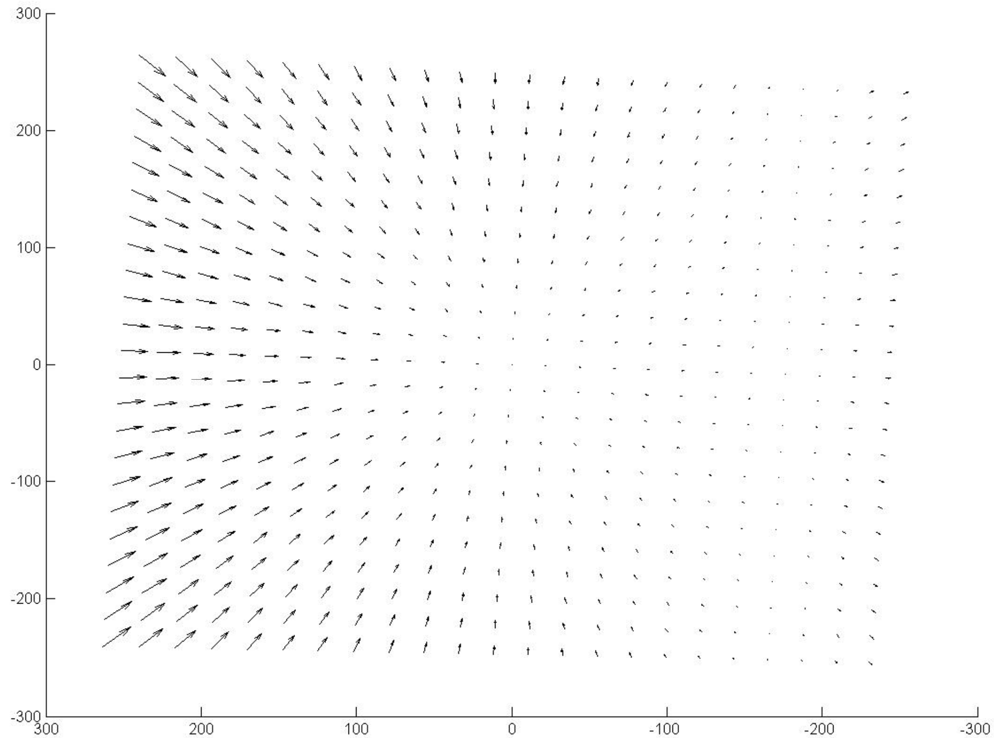


Horizontal Disparity   Vertical Disparity

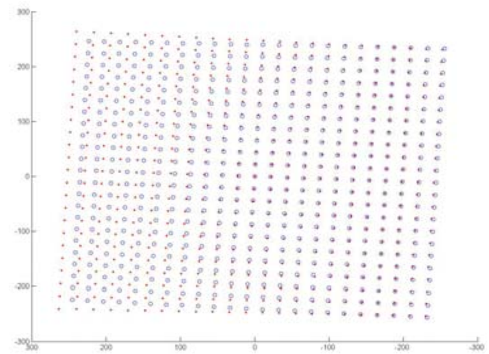
L2 law

Gaze: 10°, 30°   Distance: 30cm

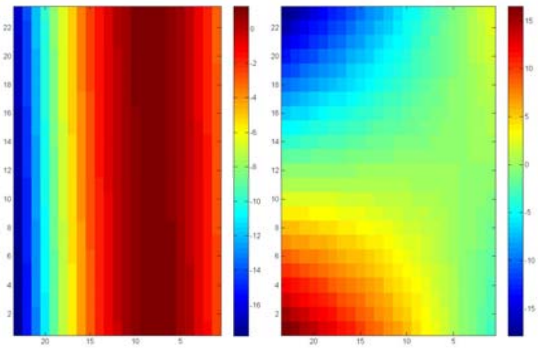
Plane: Perpendicular sight line



Disparity flow



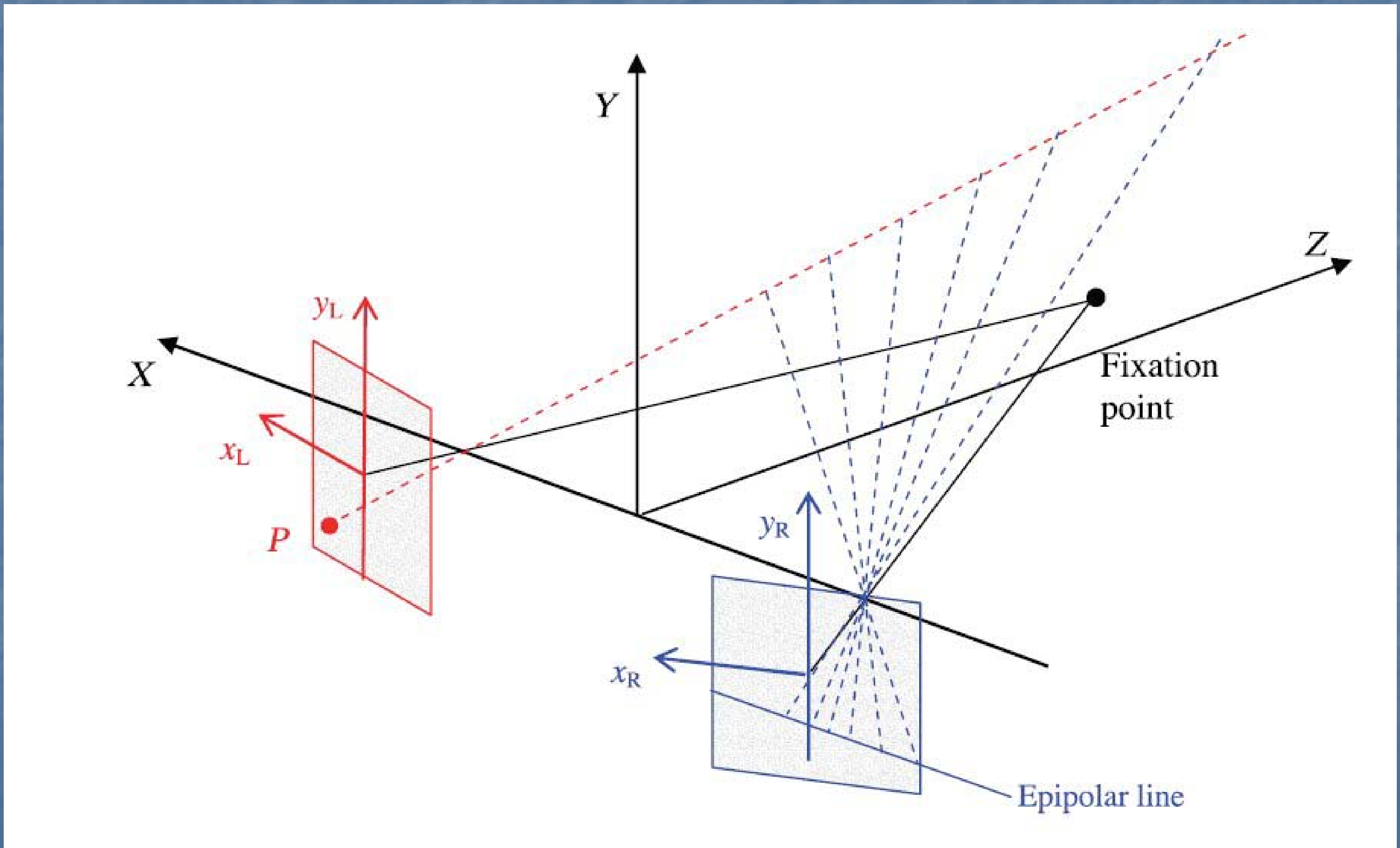
Retinal projection



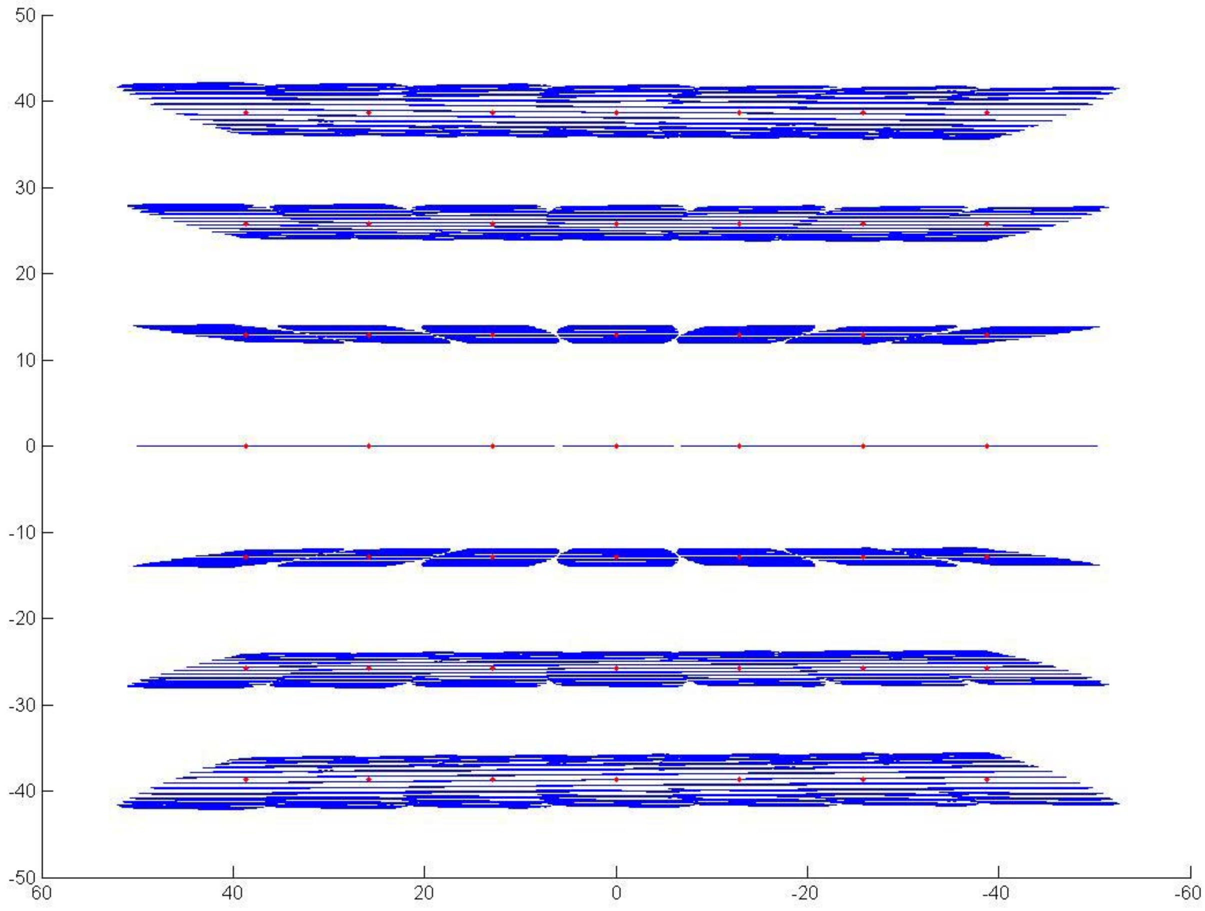
Horizontal Disparity

Vertical Disparity

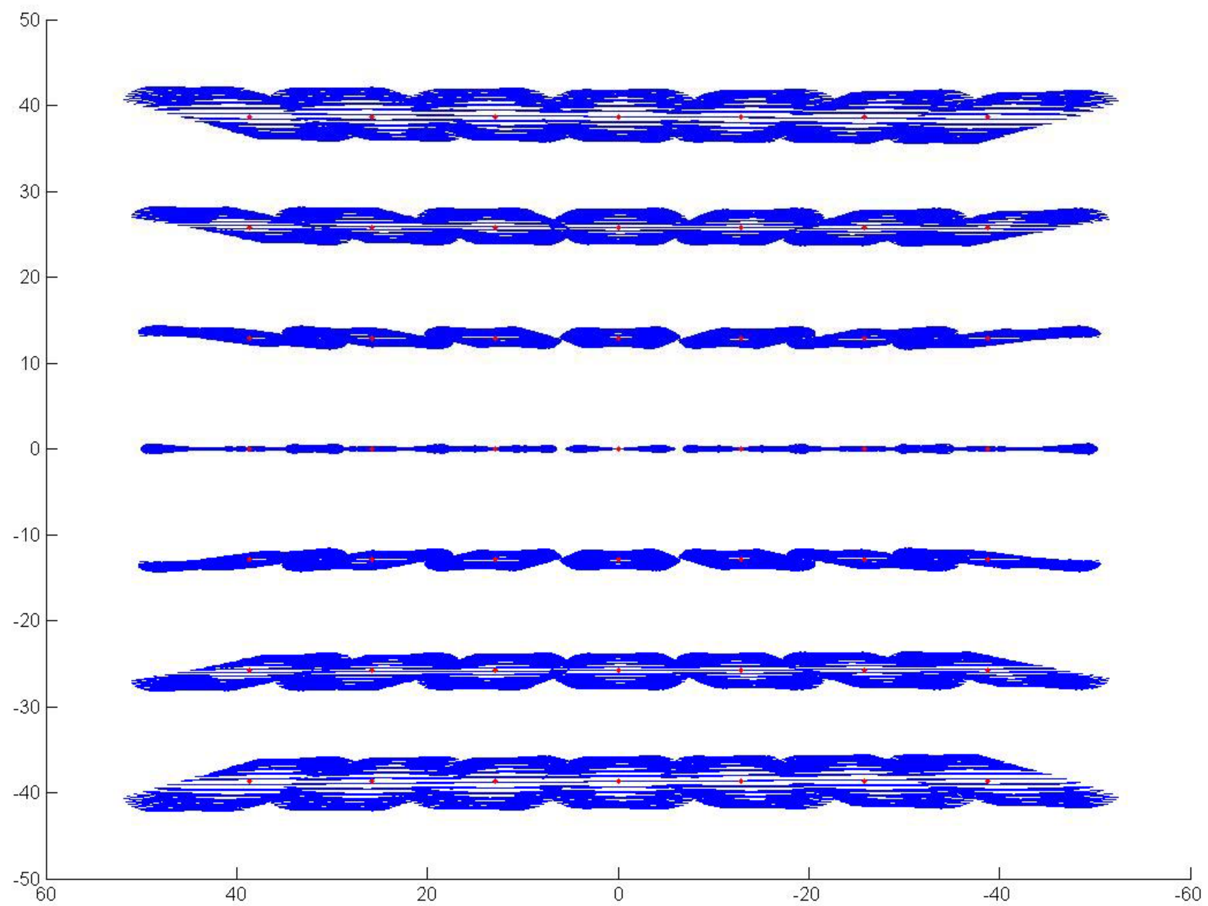
Epipolar Lines



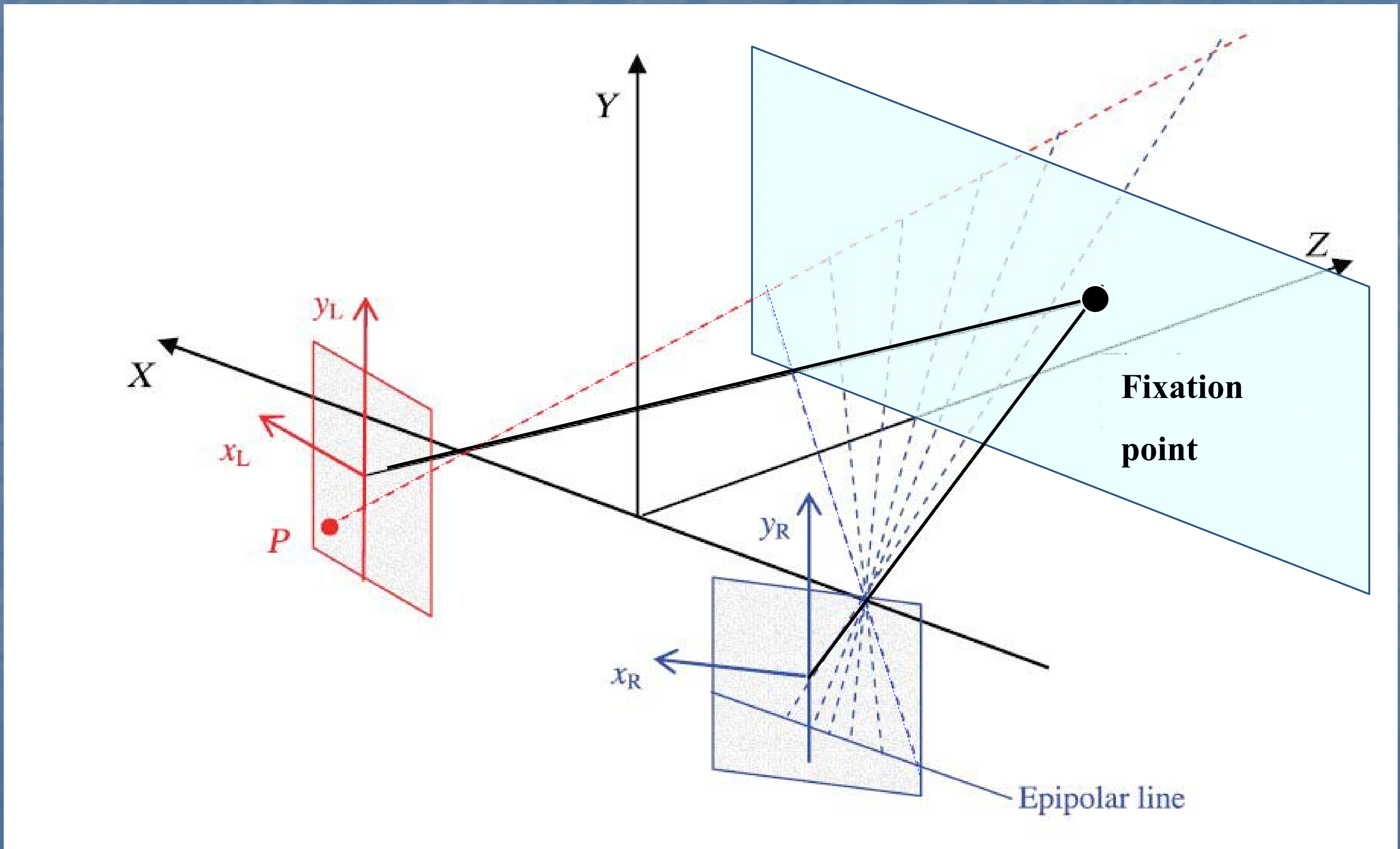
Epipolar Lines
without torsion



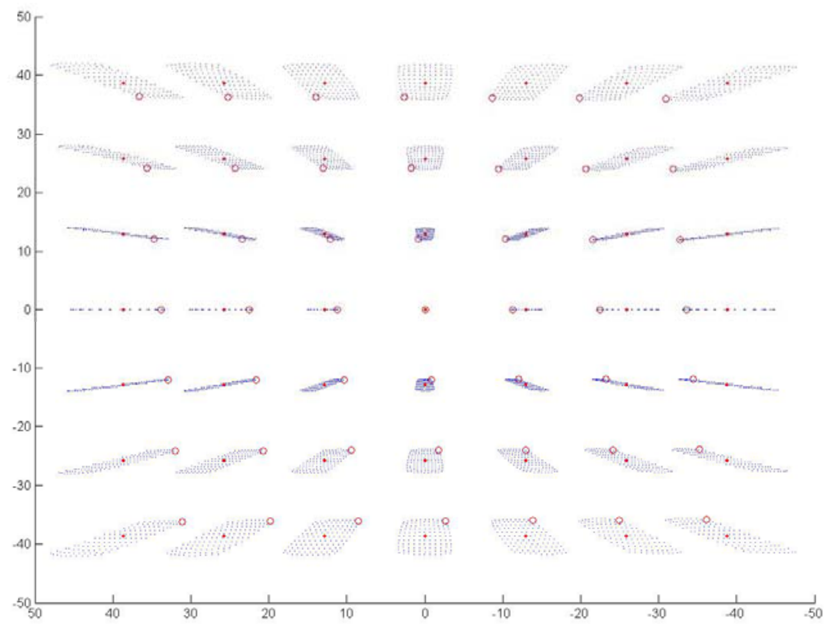
Epipolar Lines with L2 law



Epipolar Points



Epipolar Points

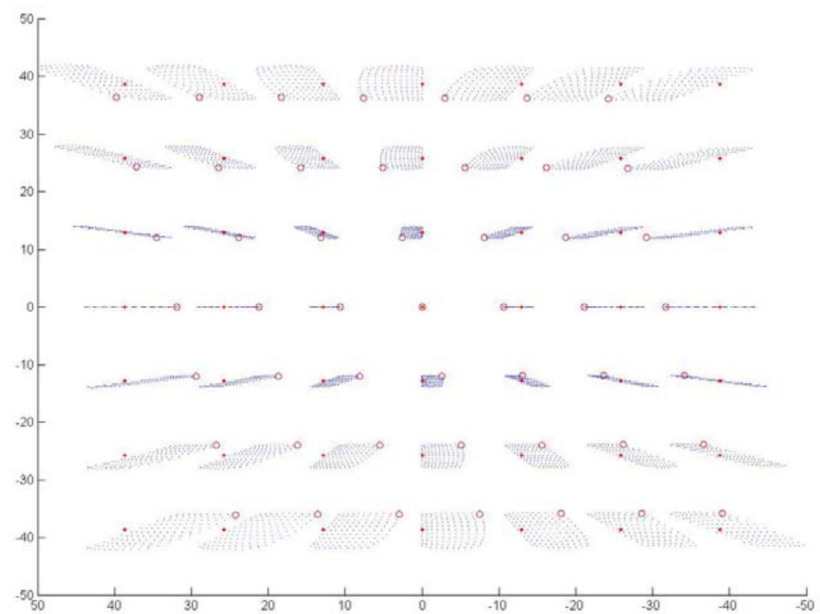


No torsion

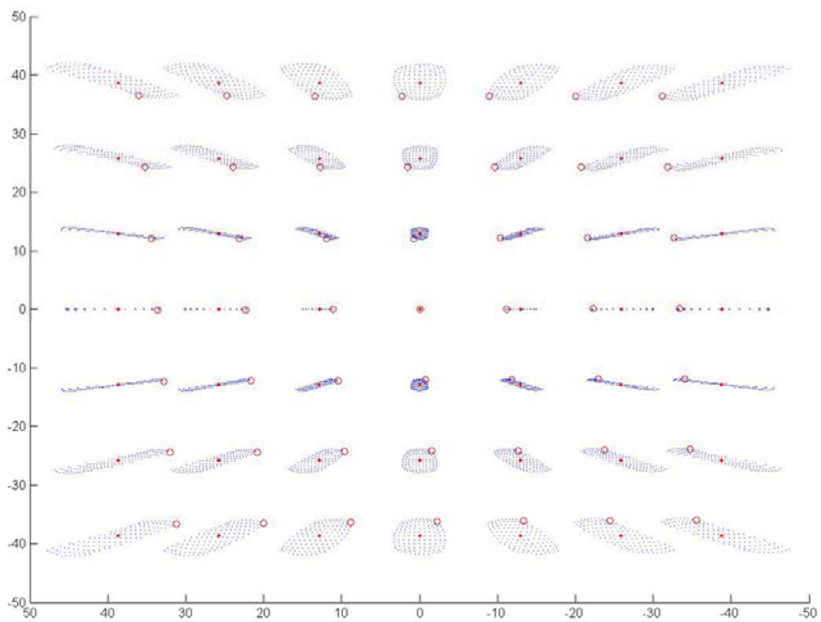
Plane: Frontoparallel

No torsion

Plane: 10°, 25°



Epipolar Points

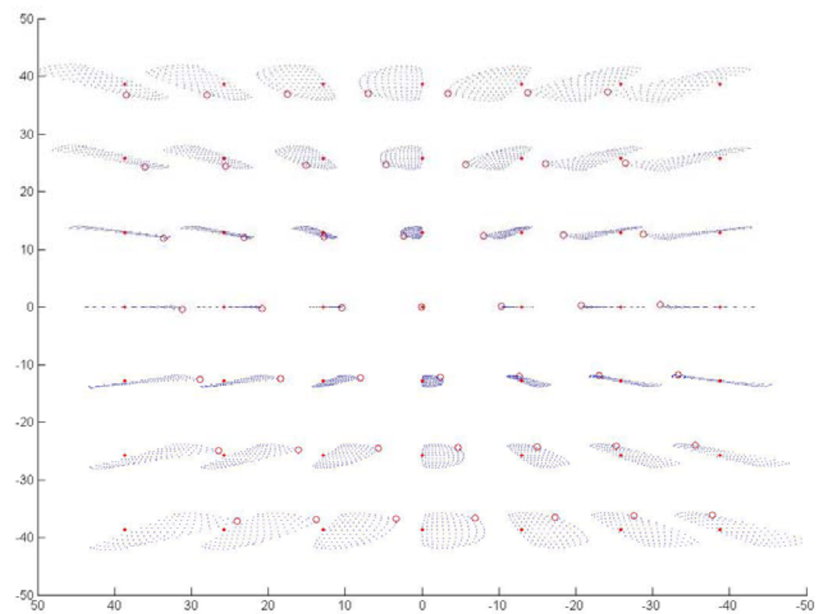


L2 law

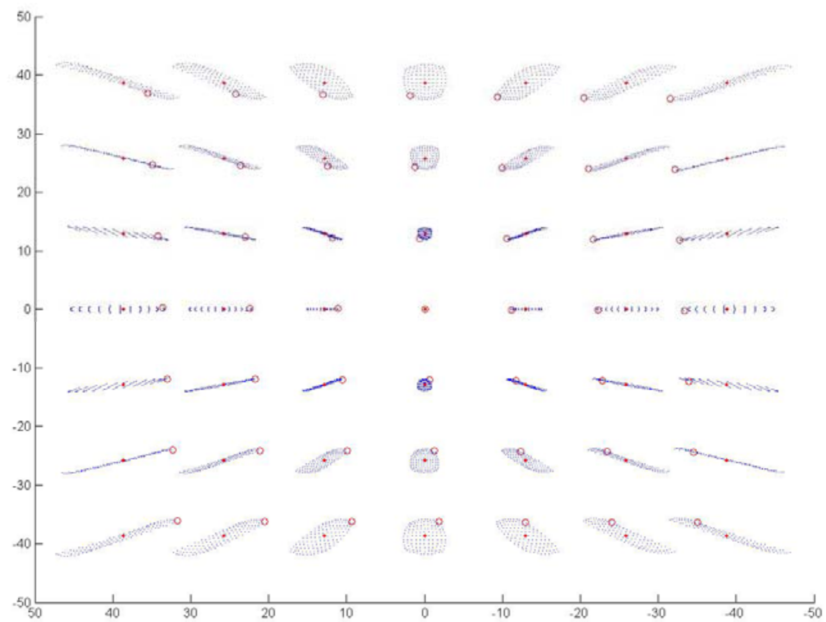
Plane: Frontoparallel

L2 law

Plane: 10°, 25°

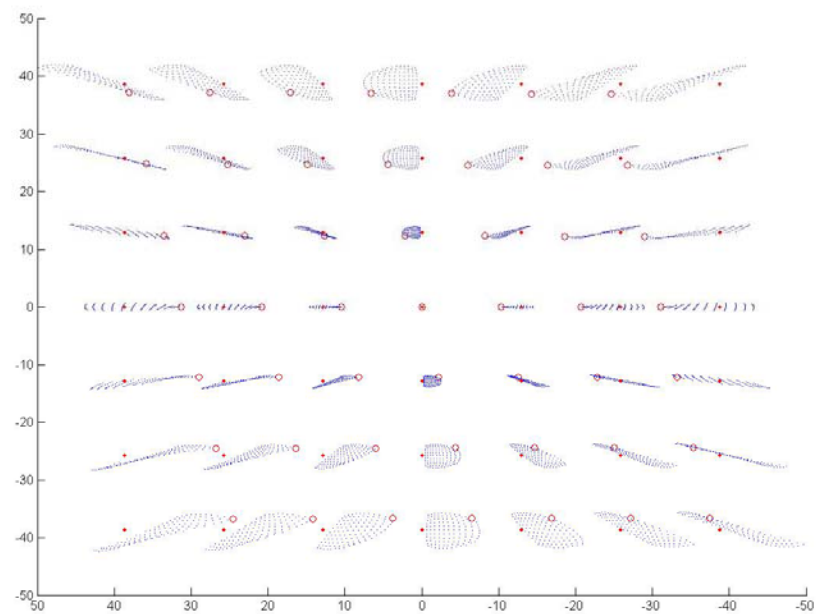


Epipolar Points



60% L2 law

Plane: Frontoparallel



60% L2 law

Plane: 10°, 25°

Open issues

- Can the torsional eyes movements play a role in the registration of the two retinic images?
- Can they be useful to reduce the mean disparity of what we are seeing?
- Are there evidences of an adaptation system that governs the eyes torsion (a Listing plane adaptation) in relation to the slant of the fixation surface?